Solving Geometric Problems Using a Non-Traditional Form of Teaching

Lucia Rumanová [1], Katarína Laššová [1]

[1] *Department of Mathematics, Constantine the Philosopher University in Nitra, Tr. A. Hlinku 1, 949 01 Nitra, Slovakia*

Abstract **– The paper focuses on innovative and creative approaches to teaching geometry in the first year of primary school. Geometry is generally not a popular part of mathematics. The findings are based on several studies investigating how students at different educational levels solve different mathematical problems. The link between geometry and the math trail is an interesting one, as it combines knowledge of geometry in the real world with solving problems directly in the field. Through the MathCityMap portal, first grade students were able to solve geometry problems created by a math trail using different measuring tools. The results of the above-mentioned outdoor activity showed that the students had gained a positive attitude towards geometry and that they had broadened their perspective on the connections between mathematics, other school subjects and everyday life.**

Keywords **– Math trail, geometry, tasks, elementary school students, problem task suggestions, guideline.**

DOI: 10.18421/TEM134-77
https://doi.org/10.18421/TEM134-77

Corresponding author: Lucia Rumanová,
Department of Mathematics, Faculty of Natural Sciences and Informatics, CPU in Nitra, Slovakia.
Email: lrumanova@ukf.sk

Received: 20 May 2024.
Revised: 31 October 2024.
Accepted: 04 November 2024.
Published: 27 November 2024.

1. Introduction

The article focuses on a developed activity called the math trail, which is a type of outdoor activity carried out in groups. It points out that even such a complex topic can be implemented in the classroom in an interesting way.

A math trail in a historical town in Slovakia was carried out with first year students. The created trail combines geometric knowledge used in a real environment with solving problems directly in the field. In such an outdoor activity group members work together, communicate, find common and best strategies for solving problems [1], [2]. The individual problems included in the math trail corresponded to their classification according to the National Education Program for Primary Education [3]. The problems in the math trail were are also inspired by the research described in [4], [5], [6].

The National Education Program for Primary Education in the area of Mathematics and Work with Information aims to build the foundations of mathematical literacy. It also aims to develop cognitive domains. Cognitive domains include knowledge as facts and procedures. It also includes the applications in which the acquired knowledge is used to solve problems in real life. Cognitive domains also include reasoning to solve more complex problems. These require a broader understanding of contexts and relationships [3].

The educational standards are divided into four areas and set out what students should know by the end of the first year of primary school:

- Natural numbers 1 to 20 and 0: the student can determine the number of elements in a group and express it as a natural number, can compare the number of elements in two groups, compare two numbers and write the result of the comparison using relational signs >, <, =, solve word problems for comparison;

- Adding and subtracting numbers up to 20: the student can add and subtract natural numbers in the number field up to 20, use the signs +, -, = and solve simple word problems;
- Geometry and measuring: the student can distinguish, name and draw planar geometric shapes, determine the position of geometric shapes in space;
- Solving application tasks and problems on developing of specific mathematical thinking: the student can sort subjects, objects and numbers according to whether they have the given property or not, decide on the truth (false) of the statement, compare two objects according to the given property [3].

Topics addressed in the form of a math trail and related outdoor activities are part of STEM education. Regarding the current situation in STEM education, there is a need for effective integration of different educational disciplines [7], which is also pointed out in the article. Research by [8] has shown the effectiveness of supporting STEM education. In STEM education, different subjects are interconnected and mathematics is often underestimated. The role of mathematics in STEM education within given problems is to use the math trail through an interdisciplinary approach as a non-traditional form of teaching.

Many mathematical problems are based on real situations. The implementation of these problems in the teaching process is very important for students, and connecting mathematical problems to the real world makes it easier for them to solve them correctly. Based on the real situation, students can better understand the meaning of mathematics teaching and learning activities [9].

Since there is a need to connect mathematics with other STEM disciplines [10], a concrete math trail was created in the park of a historical town in Slovakia. At each point of this math trail, students solved interesting geometric problems. Observations and findings from the work with students are also included in the article.

2. Materials and Methods

This article describes specific tasks that have been created as part of a mathematics trail in an urban park for first grade students in a Slovak city.

Findings are based on several studies investigating how students at different educational levels solve different mathematical problems. The negative results are also confirmed by the research reported in [11], [12]. This is confirmed by the result of research on solving mathematical problems with students from different types of schools.

The results of one of studies are presented in [13]. A math trail through a city square was constructed based on the five components of spatial ability [14] and solved by students from a vocational-technical high school. The math trail tasks focused on spatial skills, stereometry, as well as the spatial skills components of spatial perception, visualization, mental rotation, spatial relations and spatial orientation.

The results of the other research are summarized in a graph showing the percentage scores for each component of spatial ability related to task solving from the math trail. Out of five components of spatial ability, spatial perception has the highest score and spatial orientation has the lowest score, as shown in the graph in Figure 1.

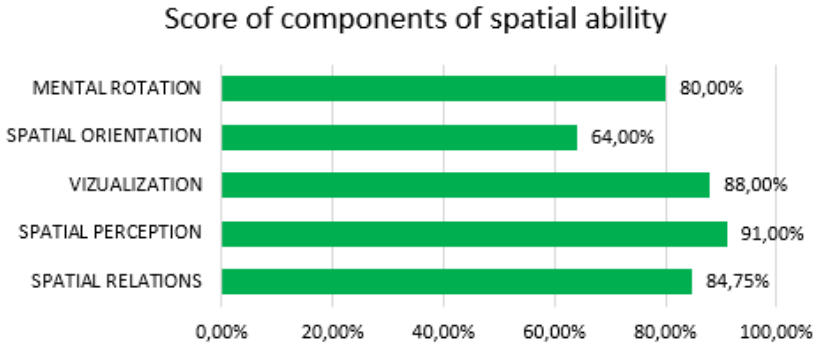

Figure 1. Score of the components of spatial ability

The results of study [15] showed that future primary school teachers had problems with geometric reasoning, with the logical system of geometry and with its proofs. These students also had problems in solving spatial ability tasks that focused on working with cubes and cube nets.

The results of the research showed that students were better able to cope with standard problems found in mathematics textbooks.

Another study [16] presented solutions to various geometry problems by students from technical secondary schools.

Students were found to have difficulty in solving various geometric problems. Similar to previous research, it was found that the students' success rate is better when they solve standard problems that are also found in mathematics textbooks.

Finally, another study is presented [17]. Students' performance on geometry problems and on the standardized Mental Rotation Test (MRT) was assessed [18]. The individual problems were solved by students from secondary vocational schools with a technically oriented course of study. Students' performance in geometric tasks was very low. For example, only 3.3% of the students correctly solved one of the problems on the cutting of a cube. Similarly, the success rate for the most technically oriented task was only 7.9%. The students' spatial ability was measured by a test. It turned out that only 6.9% of the students were successful. The results of the research showed that students need not only the knowledge, but also a sufficient amount of experience with mental rotation to be able to correctly solve various geometric problems. In addition, there was evidence that the higher the level of mental rotation students had, the more successful they were in solving the geometry problems.

Training in spatial abilities, mental rotation, geometric tasks and solving various mathematical problems is necessary from the first year of primary school. On the basis of the research, individual tasks were created in accordance with the Innovative National Education Program of primary education for the area of Mathematics and information work [3]. The math trail was then tested with 70 first grade students in one primary school in Slovakia.

3. Results

When a math trail, mentioned in this article was created, it was the first public math trail for first graders. It contained ten tasks. To carry out this math activity, students need a piece of string, a pencil and a paper. The tasks that are included in the math trail have been created in accordance with the National Education Program. The math trail consists of ten tasks that are related to real objects that are located in the city park. The math trail was solved by three first year classes. Before implementing the project, communicated with the student´s teachers. The students from each class were divided into groups of three. As the students had problems reading long texts and working on a mobile phone, each group had an adult as an advisor.

This has been done by students from math department. The individual tasks are listed in the article along with the spatial ability elements.

3.1. Task Called "Near the Gazebo"

This task is part of the Geometry and Measurement unit. In this exercise the students will recognize the basic geometric shapes of a plane. Since it is assumed that students would also know the sphere as a three-dimensional geometric figure, this figure was also included in the hint. Students can notice the difference between a circle and a sphere.

Assigned task: There is a concrete surface near the gazebo. What is the geometric shape?

Figure 2. Photo of the main object in the task "Nearby the gazebo"

Students were offered a multiple-choice answer type and the following hints:

- multiple choice: sphere / rectangle / square / circle / triangle;
- hint 1: Take a look at the shape from a greater distance;
- hint 2: It is shown in Figure 3.

Figure 3. Hint 2 in the task "Nearby the gazebo"

Sample solution to the task is that the concrete surface is in the shape of a circle. Students had no problems in solving this problem.

3.2. Task Called "Climbing Frame "

This task is included in natural numbers 1 to 20 and 0, adding and subtracting numbers up to 20 and Solving Application Tasks and Problems Developing of Specific Mathematical Thinking as well. Reading comprehension was an important part of the task.

Assigned task: The students' jungle gym is made up of red and blue metal bars attached to wooden posts. These metal bars and columns form the windows. How many windows are there where the bottom metal bar is red?

Figure 4. Photo of the main object in the task "Climbing frame"

Students were asked to answer multiple choice and given these hints:

- multiple choice: Each of the windows is made up of two metal bars and two wooden columns;
- hint 1: The window must be fully enclosed;
- hint 2: Two metal bars and two wooden columns form each window.

A sample solution is shown in Figure 5 and the result is eight windows.

Figure 5. A sample solution of the task "Climbing frame"

As expected, reading comprehension was a problem for some students. They did not realize that they were only supposed to count the windows with red bottom metal bars. Most students counted all the windows. After reading the hint, they corrected their result.

3.3. Task Called "Concrete Panel as a Fence"

This task is about geometry. It is also part of the solution of application tasks and problems that develop specific mathematical thinking. Students recognize triangles and other geometric shapes.

Assigned task: To the right of the gate is a concrete panel as a fence. How many triangles are there?

Figure 6. Photo of the main object in the task "Concrete panel as a fence"

The students were offered a multiple choice type of answer and were given these hints:

- multiple choice: 7 / 5 / 6 / 2;
- hint 1: Count only triangles that are complete;
- hint 2: A triangle cannot be cut off.

An example of how this is done is shown in Figure 7, which shows five triangles resulting.

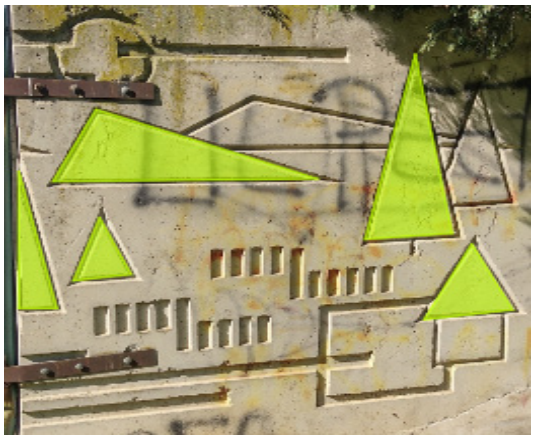

Figure 7. A sample solution of the task "Concrete panel as a fence"

The most common question asked by students was whether a superimposed triangle is a triangle. They could imagine a triangle covered by another shape. This was quite good, but their problem was finding only triangles which were fully visible. This was mentioned in the hint as well.

3.4. Task called "Educational sign Taxus Baccata "

The task is included in the following topics: Natural numbers 1 to 20 and 0, adding and subtracting numbers up to 20 and solving application tasks and problems that develop specific mathematical thinking. Students were also asked to read for understanding. As the math trail is specially designed for Slovak students, they could read the text in the learning sign in Slovak. If necessary, they could use an app to translate the text into English.

Assigned task: "Find a sign about Taxus Baccata. Read the text on the sign. In how many countries is the Taxus Baccata a protected plant? Give a number as an answer".

Figure 8. Photo of the main object in the task "Educational sign Taxus Baccata"

Students wrote down the exact value they chose to answer. The following hints were offered to the students:

- hint 1: Read only the first sentence on the sign;
- hint 2: Do not forget that in addition to the countries listed, there is another country written indirectly.

The following four countries are used to illustrate: Slovakia, Czech Republic, Poland and Serbia.

The students counted the countries called Czech Republic, Poland and Serbia while solving the task. The most common problem was that they had no idea that the country's name was Slovakia. That was reading for understanding. It was not written with the word Slovakia like the others were.

It was written with the word meaning "in our country", that means in Slovakia.

3.5. Task called "Different colored benches"

The task is included in the parts: Natural Numbers 1 to 20 and 0, Adding and Subtracting Numbers up to 20, Solving Application Tasks and Problems, and Developing Specific Mathematical Thinking. In this solution of task, the students were mainly practicing the subtraction of numbers up to ten.

Assigned task: "A blue bench and a black bench are placed next to each other in the park. Each bench has wooden planks. How many more planks does the black bench have than the blue bench?".

Figure 9. Photo of the main object in the task "Different colored benches"

Students wrote down the exact value they chose to answer. These hints were given to him:

- hint 1: Count the number of wooden boards on the black and blue bench;
- hint 2: Subtract the number of boards on the blue bench from the number of boards on the black bench.

A sample solution to the problem is three wooden boards, there are 8 wooden boards on the black bench, the blue bench has 5 wooden boards. The difference between eight and five is three. The solution to this problem was carried out without any of the typical complications.

3.6. Task called "Spring rockers"

Like the other tasks, this one is part of Geometry and Measurement, also Solving Application Tasks and Problems that Develop Specific Mathematical Thinking. The students' task was also to improve their reading comprehension.

Assigned task: "There are spring rockers in the playground. Use a string to connect the animal rocker to the motorbike rocker. What shape does the string make?".

Figure 10. Photo of the main object in the task "Spring rockers"

Students were offered a multiple-choice answer type and these hints:

- multiple choice: Square/rectangle/circle/triangle;
- hint 1: Check that you have only combined the motorcycle rocker with animal rockers;
- hint 2: It is shown in Figure 11.

Figure 11. Hint 2 in the task "Spring rockers"

An example of how to solve this problem is shown in Figure 11. and the result will be a triangle. The playground has four spring rockers. There are two animal rockers (a rooster rocker and a horse rocker) and two vehicle rockers (an airplane rocker and a motorcycle rocker). If the motorcycle rocker is connected to the rooster and horse rockers with the string, it can be seen that the string forms a triangle.

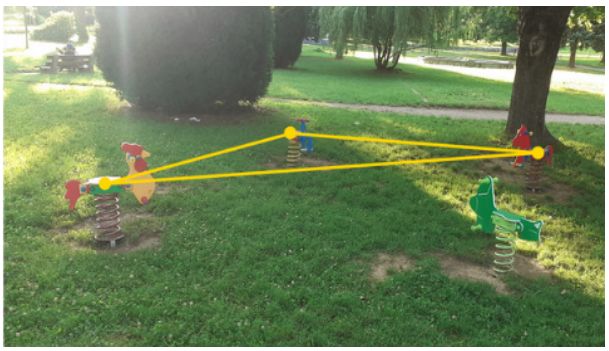

Figure 12. A sample solution of the task "Spring rockers"

3.7. Task called "Old fountain"

Geometry and Measurement is included in another set of tasks. Students recognized basic geometric planes, however, finding the right strategy for holding the string was a challenge in this task.

Assigned task: "The old fountain has four small columns. What shape does the string make if you connect the columns one by one with the string?".

Figure 13. Photo of the main object in the task "Old fountain"

Students had to write their answers in the blank spaces. These hints were given to the students:

- hint 1: It is shown in Figure 14;
- hint 2: It is shown in Figure 15.

Figure 14. Hint 1 in the task "Old fountain"

Figure 15. Hint 2 in the task "Old fountain"

A sample solution is: The string forms a square. Students had a problem attaching the string to the columns when solving this task. Each child stood on one column. This meant that the advisors also had to help hold the string, as there were fewer than four students in each group. Sometimes two groups would get together and the students would help each other out. The task was therefore a little more demanding for the students to organize and carry out.

3.8. Task called "Dandelion educational sign "

This task is contained in the Natural Numbers 1 to 20 and 0, Adding and Subtracting Numbers up to 20 and Solving Application Tasks and Problems that Develop Specific Mathematical Thinking. Students also used their knowledge of nature studies. As the math trail has been created especially for Slovak students, they would be able to read the text on the educational sign in the Slovak language. If necessary, the text could be translated into English using an app.

Assigned task: Look for a Dandelion educational sign. Read the text on the sign. How many months a year does the dandelion bloom? Give a number for the answer.

Figure 16. Photo of the main object in the task "Dandelion educational sign"

Students wrote down the exact value of their answers. Hints for students:

- hint 1: Read only the second paragraph;
- hint 2: Months of the year: January, February, March, April, May, June, July, August, September, October, November, December.

The solution is shown in Figure 16 and the answer is that the dandelion blooms for eight months. There is a title in Slovak that says it is from April to November.

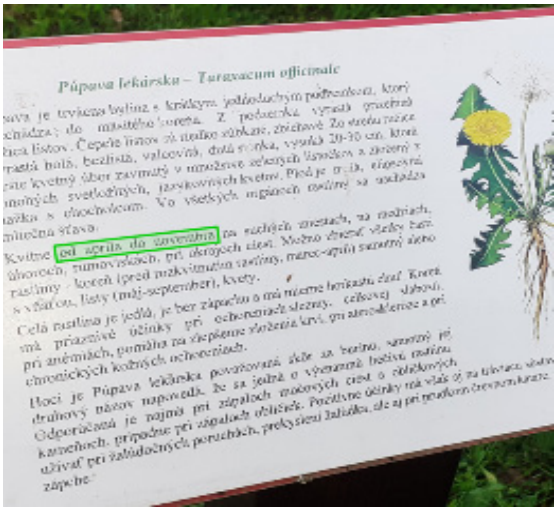

Figure 17. A sample solution of the task "Dandelion educational sign"

3.9. Task called "White gazebo "

This task is included in the Natural Numbers 1 to 20 and 0, also Adding and Subtracting Numbers up to 20.

Assigned task: There is a white gazebo in the park. The roof of the gazebo is supported by columns. There are steps leading up to the gazebo. Compare the number of posts with the number of steps. Which number is smaller? How many are there?

Figure 18. Photo of the main object in the task "White gazebo"

Students were offered multiple choice and the following hints.

Multiple choice:

- The number of columns is two less than the number of steps to climb;
- the number of steps is less than the number of columns by two;
- the number of columns is less than the number of steps by one;
- the number of steps is less than the number of columns by one;
- the number of columns and the number of steps are the same, the number of columns is less than the number of steps by zero;
- hint 1: Count the steps and count the columns;
- hint 2: Subtract the number of steps from the number of columns.

The solution to the task is zero. Since the number of columns is six and the number of steps is six, their difference is zero.

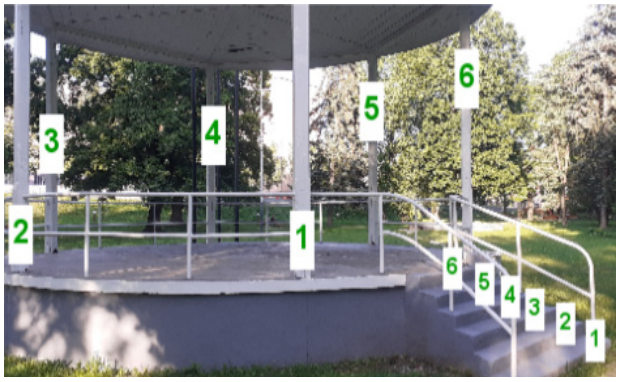

Figure 19. Sample solution of the task "White gazebo"

3.10. Task called "Distant objects"

The following two parts: Application Tasks and Problems that Develop Specific Mathematical Thinking as well as Geometry and Measurement, are part of the task. Students decide which object is further away and which is closer to them. The task develops the students´ spatial ability.

Assigned task: Stand with your back to the grand staircase at the main entrance. Look at these objects: concrete surface, gazebo, fountain, playground. Mark the true statement about each object.

Figure 20. Photo of the main object in the task "Distant objects"

Students were offered a multiple choice type of answer and these hints.

Multiple choice:

- The concrete surface is closest to me and the gazebo is the furthest away;
- the gazebo is closest to me and the playground is the furthest away;
- the fountain is closest to me, and the playground is the furthest away;
- the concrete surface is closest to me, and the playground is the furthest away;
- hint 1: "Closest to me" means the shortest distance between an object and me;
- hint 2 "The furthest from me" means the longest distance between an object and me.

An example of a solution is that the one closest to me is a concrete surface and the one furthest away from me is a playground.

4. Conclusion

Incorporating outdoor activities such as math trails has been found to help develop spatial skills in informal contexts. This benefits geometry and mathematical skills in general. It can also be said that the students' motivation increased during the experience, although the difficulty of the route was not easy for all students. Results show that students develop a more positive attitude towards geometry and broaden their perspective on the connections between mathematics, other school subjects and everyday life.

The aim of the math trail is not only to create problems, but also to show that students can experience success in solving problems and understand the importance of problem solving in practice.

The questionnaire or other summaries were not done in writing with the students. However, their reactions were monitored during the implementation of the math trail. Students experienced this mathematical activity with great emotion. They were very enthusiastic, happy and excited. From observing, it can be said that the students worked together in groups. Students thought together, they agreed together, they even corrected a wrong answer together.

The students liked the task called The old fountain the most. It was not easy to connect four points when there were only three students in a group. They liked that they had to agree with another group and help each other or ask their advisor for help.

In the Dandelion educational sign task, it was an enjoyment watching the students name the months of the year together. They also assisted and corrected a classmate when he said the months of the year in the wrong order. In addition, they counted the months of the year on their fingers. The combination of mathematics and natural history has been of great interest to the students.

Two tasks were the most common for students to have problems with (Spring Rockers and the educational sign Taxus Baccata). It was interesting that both tasks linked mathematics to reading comprehension. As the students had a problem with reading long texts, the advisors read the tasks out loud to the students in the group. So it was obvious that the students had a problem with reading comprehension.

Students mastered their mathematical knowledge very well. Working in groups helped weaker students.

The math trail is available to everyone in the MathCityMap application. Anyone can search for it by the name of "Prechádzka bánovským mestským parkom s prvákmi" or by using the code 4510408. This version is in the Slovak language.

Acknowledgements

This work was supported by the Slovak Research and Development Agency (under contract no. APVV-20-0599) and the Cultural and Educational Grant Agency of the Ministry of Education, Science, Research, and Sports of the Slovak Republic (no. KEGA 013UKF-4/2023).

References:

[1]. Čeretková, S., & Bulková, K. (2020). Mathematics trails in initial teachers' education in Slovakia. *APLIMAT 2020: Proceedings from 19th Conference on Applied Mathematics*, Bratislava, 232-237.

[2]. Lavicza, Z., Haas, B., & Kreis, Y. (2020). Discovering everyday mathematical situations outside the classroom with MathCityMap and GeoGebra 3D. *Research on Outdoor STEM Education in the digiTal Age: Proceedings of the ROSETA*, 23-30.

[3]. Štátny pedagogický ústav. (n.d.). *Matematika – primárne vzdelávanie.* Minedu.sk. Retrieved from: https://www.minedu.sk/data/att/f62/22037.f77e7d.pdf [accesed: 10 March 2024].

[4]. Rumanová, L., & Palinkásová, P. (2017). Using geometric concepts in the world of preschoolers. *Online Journal of Primary and Preschool Education* , *1*(2), 31-38.

[5]. Rumanová, L., & Záhorská, J. (2018). Nepriamo sformulované slovné úlohy v príprave budúcich učiteľov predprimárneho vzdelávania. *Acta Mathematica Nitriensia*, *4*(1), 7-12.

[6]. Uherčíková, V. & Vankúš, P. (2010). *Netradičné metódy vo vyučovaní matematiky.* Dva dny s didaktikou matematiky. Praha: Univerzita Karlova.

[7]. Yabas, D., Boyacı, H. S., & Corlu, M. S. (2020). Mathematical modelling in STEM education: A math trail using LABSTARTM. *Research on Outdoor STEM Education in the digiTal Age*, *31*.

[8]. Maass, K., Geiger, V., Ariza, M. R., & Goos, M. (2019). The role of mathematics in interdisciplinary STEM education. *Zdm*, *51*, 869-884.

[9]. Blum, W. (1993). Mathematical Modelling in Mathematics Education and Instruction. *Teaching and Learning Mathematics in Context*; 3–14.

[10]. Wang, H. H. (2012). *A new era of science education: Science teachers' perceptions and classroom practices of science, technology, engineering, and mathematics (STEM) integration*. University of Minnesota.

[11]. Rumanová, L., & Záhorská, J. (2019). Mistakes in Solving Selected Geometry Problems. *Acta Mathematica Nitriensia*, *5*(1), 23-29.

[12]. Pavlovičová, G., Bočková, V., & Rumanová, L. (2023). Analysis of the Van Hiele test solutions of 9the graders in Slovakia. *ICERI2023 Proceeding, IATED,* 2573-2579.

[13]. Laššová, K., & Rumanová, L. (2023). Engaging STEM Learning Experience of Spatial Ability through Activities with Using Math Trail. *Mathematics*, *11*(11), 2541.

[14]. Maier, P. H. (1996). Spatial geometry and spatial ability–How to make solid geometry solid. *Selected papers from the Annual Conference of Didactics of Mathematics*, 63-75.

[15]. Pavlovičová, G., Bočková, V., & Laššová, K. (2022). Spatial ability and geometric thinking of the students of teacher training for primary education. *TEM Journal*, *11*(1), 388.

[16]. Laššová, K., (2023). *Analýza riešení stereometrických úloh u žiakov stredných odborných škôl s technickým zameraním*. [Rigorous Thesis]. Constantine the Philosopher University in Nitra. Faculty of Natural Sciences and Informatics.

[17]. Laššová, K., (2024). *Zobrazovanie priestoru a jeho aplikácie v strednom odbornom vzdelávaní.* [Dissertation Thesis]. Constantine the Philosopher University in Nitra. Faculty of Natural Sciences and Informatics.

[18]. Vandenberg, S. G., & Kuse, A. R. (1978). Mental rotations, a group test of three-dimensional spatial visualization. *Perceptual and motor skills*, *47*(2), 599-604. Doi: 10.2466/pms.1978.47.2.599.