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Application of the Beating Method to Define the Electroencephalogram α–Rhythm Frequency Content

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Abstract **– Among plenty of the brain oscillation electrical patterns, the spindle oscillations in the alpha-rhythm band has main functional significance, in particular in emotional, memory, motivation and attention processes. At the same time, signal processing techniques well developed in radio engineering are not used in electroencephalogram (EEG) processing. For the first time the beating method was applied to electroencephalograms. It is shown that the beating method allows the determination of α–rhythm frequencies, which are not always resoluted using Furrier transform. EEG is non-stationary process that's why instantaneous frequency is a very important characteristics of EEG. The calculation of instantaneous frequency values was carried out based on the Hilbert transform and intersection detector method. A comparative analysis of electroencephalograms frequency content obtained by Furrier transform, Hilbert transform and Zero-level intersection detector method was carried out.**

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1. Introduction

For the first time, the term alpha-rhythm was introduced by G. Berger in 1929, and a number of researchers called it Berger's rhythm.

Alpha rhythm (English alpha rhythm, α-rhythm) represents biopotentials of the brain with a frequency of 8 - 13 Hz, and amplitude of 30-100 µV [1], [2], [3].

An alpha rhythm with maximum amplitude is registered in the parietal-occipital region (projection of the sensory cortex of the brain) with closed eyes at rest in 85-95% of healthy adults. Towards the frontal lobes of the brain, the amplitude and representation index of the alpha rhythm decrease.

The phenomenological characteristic of the alpha rhythm is modulations expressed in the alternating increase and decrease of the amplitude of the waves with the formation of characteristic "spindles", the duration of which most often varies from 2 to 8 s. There is a concept of regularity of the alpha rhythm, when the periods of the waves differ by no more than 0.5 Hz [1], [4].

Currently, a distinction is made between alpha rhythm and alpha activity. Alpha activity usually consists of single alpha waves with a duration of 80 to 125 ms and can be recorded in any brain structure. The alpha rhythm is defined as regular wave activity with a frequency of about 10 Hz and is recorded mainly in the parieto-occipital regions. However, it has been noted that there may be different cases of alpha rhythm distribution; gradual wave-like propagation of alpha waves from the occipital regions of the cortex to the frontal; earlier appearance of the alpha rhythm in the frontal areas; propagation of alpha waves from the frontal to occipital areas of the brain.

This gives reason to believe that the alpha rhythm may primarily arise in different areas of the human cerebral cortex, although the focus of its activity is tied to the occipital region of the cortex. The latter is confirmed by studies of the electrical activity of the human brain with visual impairment, which showed a direct dependence of the alpha rhythm on visual afferentation [3], [5].

The next quantitative characteristic is the alpha activity index. Up to 25% alpha activity is considered low, up to 50% - medium, high - more than 70%. Alpha activity is considered normal if it is recorded in the parieto-occipital region with an index of at least 50% [1], [5].

It has been established that the amplitude and frequency of biopotentials recorded on the surface of the scalp are the result of a sequence of processes of inhibition and excitation of postsynaptic potentials of pyramidal neurons of the cortex, which depend on the influence of cortical modules, cortico-thalamocortical interactions and the inhibitory influence of thalamic reticular fibers. In 1968, P. Andersen and S. Andersson came to the conclusion that the thalamic nuclei are the primary generator of all types of rhythmic fusiform brain activity [1], [6].

Frequency-amplitude characteristics of alpha activity are indicators of the functional state of the cerebral cortex. A change in the level of functional activity of the brain (decreasing or increasing the degree of activation) leads to depression or the appearance of alpha waves on the EEG. If, against the background of quiet wakefulness, activating influences are given (increasing the level of attention, cognitive load, etc.), a blockade of alpha activity is observed. Against the background of a drowsy state (decreased tone of the cerebral cortex), the same activating influences cause a burst of alpha activity [5], [6], [7]. The frequency of alpha activity varies markedly from person to person, but it is extremely stable for each individual. A change in the frequency of alpha activity in the brain is determined by a change in the calcium concentration in the cytoplasm of the neuron. Moreover, the structural organization of calcium channels, being genetically determined, determines this frequency. The heritability of alpha activity frequency has been proven by many studies. This indicator is an indicator of individual typological differences. It has been shown that individuals who differ in the frequency of the maximum peak of alpha activity are characterized by different behavioral strategies. Individual variability in the frequency of the maximum peak of alpha activity has been established depending on age, neurohumoral status, the nature of the psychopharmacological effect, the severity of fatigue processes, fluency in performing cognitive and psychomotor tasks, as well as the degree of proficiency in professional skills [5], [8], [9].

Another important characteristic is the modulation of alpha oscillations. It has been shown that EEG spindles are the result of the interference of spontaneous and evoked activity or the interaction of several self-oscillatory devices existing within the frequencies of the alpha range. The interaction of a number of such oscillators leads to amplitude and frequency-phase modulation, and they have clearly defined nonlinear characteristics. Therefore, the alpha rhythm spindles are irregular and their duration varies over a fairly wide range. In this case, the connections between individual oscillators within a given frequency range are obviously such that the system has not yet entered the stochastic mode, but has already left the periodic one [1], [5], [10].

The formation of fusiform alpha oscillations involves volley and tonic types of impulses. These two types of neuronal discharges - tonic and volley - effectively transmit information to the cortex, but in a different sequence. If the tonic type allows for accurate “linear” encoding of information, then the salvo discharge, depending on the value of the signal-to-noise ratio, is used to detect subtle changes in the environment. Due to the fact that thalamic neurons have the ability to generate potentials of two types, the generated total alpha waves have a spindle-shaped appearance and have the property of autorhythmicity [10], [11].

Burst discharges cause cortical activation and act as “wake-up stimuli”, which is effective for the execution of automatic actions. As soon as attention increases, tonic impulse mechanisms are activated to activate the transmission of information along the thalamo-corticothalamic circuits. Fusiform activity is recorded during quiet wakefulness or "awaiting attention." The same mechanism of spindle formation is observed during spontaneous spindle oscillations in the stage of superficial slow-wave sleep. Until recently, it was assumed that sleep spindles and alpha spindles were completely different phenomena. In recent years, ideas have been developing that, on the one hand, spindles are associated with inhibitory processes in the thalamus and can be considered as an “input mechanism” for blocking the process of processing information from the outside world, and on the other hand, on the contrary, they play the role of a “probing” or the controlling mechanism of the body's connection with the environment. Thus, the presence of well-defined alpha spindles on the EEG indicates that the adaptive mechanisms of ascending and descending control are well-functioning [12], [13].

The hypothesis of “Periodization of Inhibition” has been proposed, which explains the principles of formation of the amplitude of alpha oscillations.

Mierau, A. et al [14] believes that the amplitude of alpha oscillations reflects rhythmic changes in the phases of minimum and maximum inhibition and that desynchronization of these waves reflects the status of relatively high excitability, while synchronization reflects the status of inhibition [14].

An increase in amplitude leads to precise periodicity of neural activity, since it is carried out by discharging many neurons simultaneously, which takes a certain short time. Inhibition resulting from tonic impulses will not accelerate the time of neural activity, but will affect the "silence" of all cells and completely block the information processing process. In contrast, inhibition resulting from volley impulses may help speed up information processing. Accordingly, the increased amplitude of oscillations will indicate not activation processes, but inhibition, when most or all of the main cells turn out to be "silent", and oscillations occur due to the discharge of many neurons simultaneously and take only a short time. It is important to note that during a state of inhibition, cognitive processes continue to occur, such as top-down information processing. The process of descending control is the process of inhibition of functions with the participation of attention, which allows highly selective focusing of task performance, using the processes of inhibition of irrelevant areas of the brain, so as not to interfere with the work of relevant ones [14].

The next functional feature of alpha oscillations is a decrease in the amplitude of alpha waves in the occipital region of the cortex in response to light stimulation. It has been proposed to use the term "activation" to denote the reactivity of the alpha rhythm to visual stimulation. Activation or response to eye opening is considered as a transition from a state of rest to a state of activity, that is, it characterizes the speed/inertia of inhibition processes or the active process of information processing. It has been shown that the duration of alpha amplitude suppression when opening the eyes is one of the informative signs of the stability of the activation reaction and, together with the depth of desynchronization, correlates with the effectiveness of intellectual activity. It has been proposed to use indicators of amplitude suppression in response to eye opening as individual typological signs related to both general adaptability and the formation of changes in cognitive activity [15], [16].

Thus, the study of the characteristics of alpha activity will make it possible to identify the most informative indicators for assessing the functional activity of the brain, both under normal conditions and during the development of various pathological conditions.

Obviously, any specific measurement of a characteristic, such as amplitude or frequency, cannot be considered a complete estimate of the α-rhythm.

There for, the determination of behavior and adaptation mechanisms, analysis of the entire series of individual parameters of the EEG α-activity is required to be carried out.

2. Basic Approaches to Determining α–rhythm Frequency Content

Analyzing the alpha rhythm in EEG is crucial for understanding brain activity. The alpha rhythm is associated with states of relaxation and wakefulness. By studying these patterns, researchers can gain insights into various neurological conditions, monitor cognitive states, and even assess the effectiveness of treatments for disorders such as epilepsy and depression.

2.1. Beating Method

Beating method (BM) is the result of a linear transformation. A system of oscillatory circuits will be considered as a physical model of a linear system with beats [17].

Let us consider the response of such a system to harmonic voltage. If contour has no losses, transient current is a combination of two harmonic oscillations having close frequencies and approximately equal amplitudes. As a result of the addition of these oscillations in the circuit, so-called beating appears:

$$i \approx -I_0 \sin(\omega t - \varphi) + I_0 \sin(\omega_0 t - \varphi) = 2I_0 \sin\left(\frac{\omega-\omega_0}{2} t\right) \cos < \left(\frac{\omega-\omega_0}{2} t + \varphi\right); \quad \omega > \omega_0.$$

From the resulting expression it follows that the amplitude of the current in contour changes slowly over time according to the law $\left|\sin\left(\frac{\omega-\omega_0}{2} t\right)\right|$, and the frequency of current oscillations is $\omega \approx \frac{\omega+\omega_0}{2}$.

The curve characterizing the change in oscillations over time is called an envelope. In the considered beat mode, envelope is determined by the function [17]:

$$I(t) = 2I_0 \left|\sin\left(\frac{\omega-\omega_0}{2} t\right)\right|.$$

If the oscillation frequency of the source differs significantly from the resonant frequency of the contour, the transient nature of the circuit changes.

The nonlinear electronic component is affected by the signal voltage $\boldsymbol{u_0}$ and the harmonic voltage received from the generator called heterodyne. The signal voltage spectrum consists of frequencies $\boldsymbol{\omega_{c1}, \omega_{c2}, \dots, \omega_{ck}, \dots, \omega_{cn}}$ [17]. Unlike amplitude modulation, in which the frequencies of the applied voltages are sharply different, the heterodyne frequency differs little from any of the signal frequencies $\boldsymbol{\omega_{ck}}$

$$\omega_{ck} = \Delta\omega + \omega_g.$$

Here $\Delta\omega \ll \omega_g$.

Let us consider the beats that occur in a nonlinear system. The output voltage of a nonlinear electronic component contains many oscillations of different frequencies. In particular, the output signal will include difference frequencies $\omega_{ck} - \omega_g$ and sum frequencies $\omega_{ck} + \omega_g$.

If the filter only passes difference and sum frequencies, then the signal spectrum is converted into two spectra. These spectra are shifted to the region of lower (difference) and higher (total) frequencies.

It is assumed that the shape of the spectral function of the signal remains unchanged, that is, there is no distortion. This ensures the selection of the appropriate type of nonlinearity. The most common use is to shift the spectrum to lower frequencies. Therefore the heterodyne filter must pass only difference frequencies.

Let us look at the heterodyne process in more detail. Let us consider the input signal in the form of a harmonic oscillation with frequency ω_c. The sum of the heterodyne and signal voltages is

$$u = U_g \cos(\omega_g t) + U_c \cos(\omega_c t).$$

The difference between the signal frequencies and heterodyne $\Theta = \omega_c - \omega_g$ is very small: $\Theta \ll \omega_g$.

As a result of the addition of two oscillations that differ slightly in frequency, beats are formed. The envelope amplitude changes with difference frequency Θ ranging from $U_{min=}U_g - U_c$ to $U_{max=}U_g + U_c$. [17].

The resulting oscillation can be thought of as a frequency oscillation ω_g, initial phase of which periodically fluctuates with frequency Θ within $\mp\varphi_{max}$. The maximum phase deviation is determined from the equation

$$sin\varphi_{max} = U_c/U_g.$$

On the other hand, a change in the initial phase of an oscillation is inextricably linked with a change in its frequency. Therefore, it can be argued that the resulting oscillation, having a constant initial phase, changes over time with frequency

$$\omega = \omega_g + d\varphi/dt.$$

Amplitude of total oscillation equals to

$$U = \sqrt{U_g^2 + U_c^2 - 2U_g U_c cos\beta},$$

From here-it follows that:

$$U = U_g\sqrt{1 + \frac{U_c^2}{U_g^2} + 2\frac{U_c}{U_g}cos\theta t}.$$

This expression represents the law of changes in amplitude in time, that is, the law of envelope changes.

$$\frac{U}{U_c+U_g} = \sqrt{\frac{1+\frac{U_c^2}{U_g^2}+2\frac{U_c}{U_g}cos\theta t}{1+\frac{U_c^2}{U_g^2}+2\frac{U_c}{U_g}}}.$$

The shape of the signal envelope depends on the relationship U_c/U_g. When $U_c/U_g \ll 1$, envelope is close to a sinusoid. The greater the ratio U_c/U_g, the more the shape of the envelope differs from sinusoids [17].

Let us consider two particular cases in more detail.

1. Let the signal voltage be much less than the heterodyne voltage $U_c \ll U_g$. It follows that the angle $\varphi_{max} \approx 0$, that is, it can be assumed with sufficient accuracy that the frequency of the resulting oscillation is equal to the frequency of the local oscillator, i.e. $\omega \approx \omega_g$ [17].

Further, since$U_c/U_g \ll 1$, then

$$U \approx U_g\sqrt{1 + 2\frac{U_c}{U_g}cos\theta t} \approx U_g(1 + \frac{U_c}{U_g}cos\theta t),$$

Denoting $\frac{U_c}{U_g} = m$, it follows that $U = U_g(1 + mcos\theta t)$, i.e. the envelope has a harmonic shape. The beats in this case are similar to an oscillation modulated in amplitude by a harmonic signal of frequency θ with modulation index m [17].

2. Signal and heterodyne voltages are equal: $U_c = U_g$.

Because $2\varphi = \pi - \beta$ and $\beta = \pi - \theta t$, then $\varphi = \theta t/2$, and the frequency of the resulting oscillation is equal to $\omega = \omega_g + \frac{\theta}{2} = \frac{\omega_g + \omega_c}{2}$.

So, when $U_c = U_g$, the frequency of the resulting oscillation is equal to half the sum of the frequencies of the added oscillations [17].

From here, at $U_c = U_g$ the beat amplitude is:

$$U \approx U_g\sqrt{2(1 + cos\theta t)} = 2U_g cos\frac{\theta t}{2} = 2U_g cos\frac{\omega_c - \omega_g}{2}.$$

I. e. the beat amplitude changes from zero to $2U_c = 2U_g$.

When beats act on nonlinear resistance, they can be converted into oscillations of the difference frequency θ, which is called the beat frequency. For this, it is enough that the nonlinear electronic component has the properties of an amplitude detector. Indeed, the amplitude of the beats changes over time with frequency θ. Thus, the process at the output of the amplitude detector contains oscillations of frequency θ [17].

At first glance at the alpha spindles, it is noticeable that they look like beating (Figure 1).

The mathematical theory of the beating process that occurs during the addition of harmonic signals was developed in [18], [19], and then in [20]. According to the results of these studies, when summing harmonic signals that are close in frequency, oscillations arise that are modulated in amplitude, frequency and phase. Resultant signal can be expressed as

$$\boldsymbol{s(t)} = A_1 cos(2\pi f_1 t) + A_2 cos(2\pi f_2 t) \quad (1)$$

Frequencies difference is extremely small

$$|f_1 - f_2| << f_2.$$

Envelope amplitude varies within the limits of $|A_1 - A_2|$ to $A_1 + A_2$. Therefore, envelope shape depends on A_1/A_2 ratio. Envelope is similar to sine when one of the beatings has small amplitude.

Beating of the sum of two harmonic signals is shown in Figure 2.

Let us apply the beating method to alpha spindles of the typical EEG obtained during testing of the awake adult.

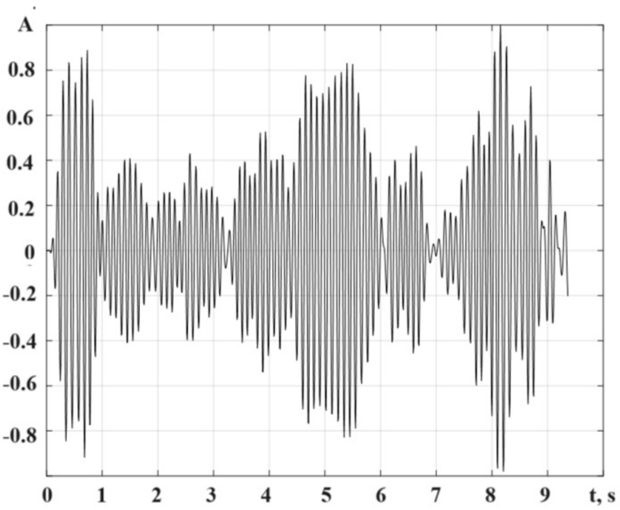

Figure 1. Alpha-rhythm of the awake adult

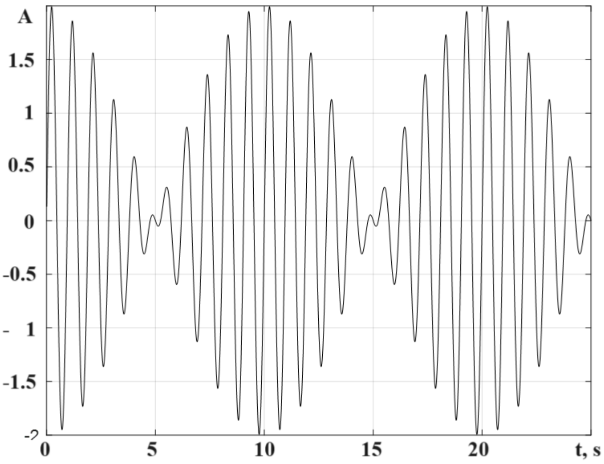

Figure 2. Beating of the sum of two harmonic signals:

f1 = 1 Hz; f2 = 1.1 Hz

As can be seen, in this case, the signal can be considered as a sequence of radio pulses, for which carrier is defined by [21].

$$f_C \simeq (f1 + f2)/2 \quad (2)$$

and frequency envelope correspondenly equals

$$f_E \simeq (f1 - f2)/2 \quad (3)$$

These equalities will be exact in the case when the amplitudes of the resulting oscillations are identically equal. It can be seen that the amplitude spectrum in Figure 3 indeed corresponds to an amplitude modulated oscillation whose lateral components are located quasi-symmetrically coupled to the carrier. It is obvious as well, that the resolution of the beating frequencies is impossible.

Let us now consider an individual α-spindle (Figure 4), and define its carrier and envelope frequency. Then the spectrum of the considered oscillation is determined using Furrier transform (FT) (see Figure 5).

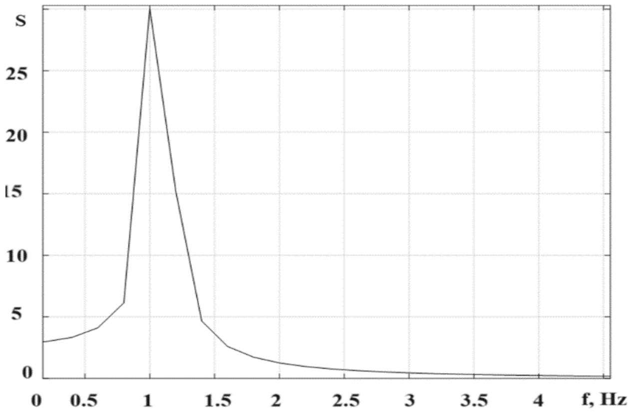

Figure 3. Amplitude spectrum of the sum of two harmonic signals: f1 = 1 Hz; f2 = 1.1 Hz

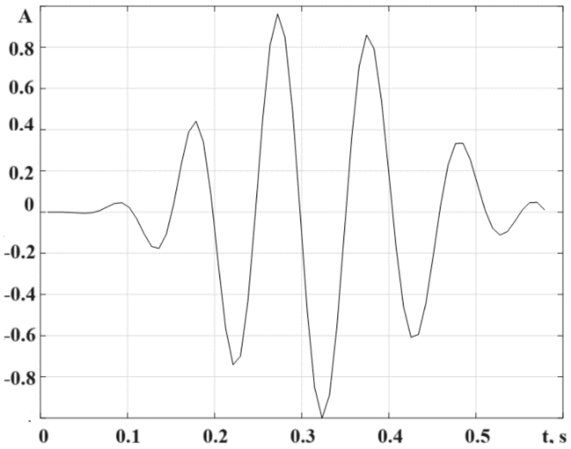

Figure 4. EEG alpha spindle (signal 1)

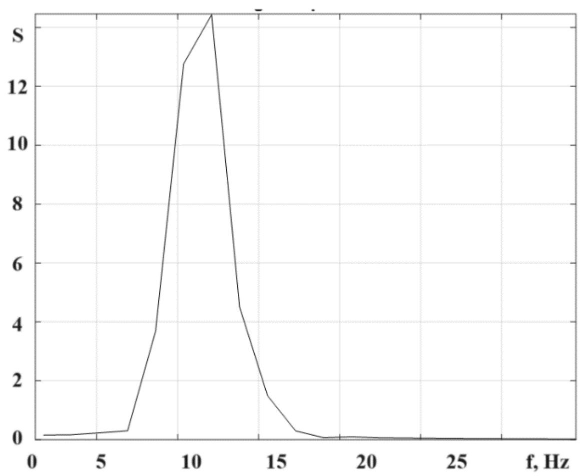

Figure 5. Amplitude spectrum (signal 1)

The appearance of a well pronounced asymmetry and an additional maximum in the spectrum allows both to observe the presence of two harmonics and to estimate their values. Thus, comparing the results of the beating method and the Furrier transform shows that, unlike beating method, the Furrier transform does not provide resolution of nearest frequencies in all cases considered.

2.2. Hilbert Transform

EEGs refer to non-stationary processes, so let us consider a number of other methods that are used in the analysis of non-stationary processes [22], [23].

One of the most accurate methods of analysis of non-stationary processes is Hilbert transform (HT).

When modulating and analyzing signals the Hilbert transform and the associated concept of analytic signal are of great practical importance [24], [25].

Let there be a signal *s(t),* then the orthogonal complement of the signal *s(t)* is called signal s_{orth} such that

$$\int_{-\infty}^{\infty} s(t)s_{orth}(t)=0$$

This implies that что s_{orth} is not identically equal to zero. allows us to calculate the orthogonal complement of the signal *s(t)*:

$$s_{orth}=\int_{-\infty}^{\infty}\frac{s(\tau)}{\pi(t-\tau)}\,d\tau.$$

From this expression it can be seen that Hilbert Transform is the result of signal convolutions *s(t)* with function $h(t)=\frac{1}{\pi t}$, which is called Hilbert transform kernel. Thus, the Hilbert transform kernel is an impulse response of a linear filter, the output of which is formed by an orthogonal complement of the input signal. A filter with impulse response $h(t)=\frac{1}{\pi t}$ is called a Hilbert filter. The frequency response of the Hilbert filter is purely imaginary and is equal to:

$$H(\omega)=\frac{-j}{\pi}\pi*sign(\omega)=-jsign(\omega)=\begin{cases} j \text{ if } \omega<0; \\ 0 \text{ if } \omega=0; \\ -j \text{ if } \omega>0. \end{cases}$$

It can be concluded that the Hilbert filter is an ideal phase shifter. Like any ideal filter, the Hilbert filter does not meet the condition of physical realizability. In addition, the Hilbert filter removes the DC component of the signal.

Thus, the Hilbert transform in the frequency domain can be written as:

$$S_{orth}=H(\omega)S(\omega).$$

The original signal can be obtained via inverse Hilbert transform:

$$s(t)=\int_{-\infty}^{\infty}\frac{s_{orth}(\tau)}{\pi(t-\tau)}d\tau.$$

A complex signal of this type is called an analytic signal:

$$z(t)=s(t)+js_{orth}(t).$$

Consider the spectrum of the analytic signal:

$$Z(\omega)=\begin{cases} 0 \text{ if } \omega<0; \\ S(0) \text{ if } \omega<0; \\ 2S(\omega) \text{if } \omega>0. \end{cases}$$

Thus, the analytic signal spectrum is nonzero only at positive frequencies, and in the negative frequency region the analytic signal spectrum is equal to zero.

This property of the analytic signal is widely used in the formation of single-sideband modulation. In addition, the analytic signal can be used to build orthogonal complement.

Let us derive a Furrier transform from the original signal, zero the spectrum in the negative frequency range, double the spectrum in the positive frequency range, then derive an inverse Furrier transform and get an analytic signal. Then, from the analytic signal let us derive the original signal and its orthogonal complement. A similar procedure can be implemented digitally using z-Transform.

Let us determine the instantaneous frequency using Hilbert transform [24], [25].

The Hilbert transform is based on the representation of a random process $s(t)$ in the form of $s(t) = A(t)\cos\varphi(t)$ [26], [27]. Thus, the random process s(t) is considered as a harmonic fluctuation modulated in amplitude and in phase (or frequency) by the stochastic functions $A(t)$ and $\varphi(t)$. The calculation of these functions lets one to observe the changes of signals amplitude and phase in time. Simultaneously, changes of instantaneous frequencies are investigated. Considering the frequency as a derivative of the phase of the oscillatory process, then the instantaneous frequency is determined by the following equality: $f(t) = 1/2\pi(d\varphi/dt)$.

Therefore, each of instantaneous values of the signal has its own values of amplitude and frequency. In Figure 6 is shown the instantaneous frequency dependence on time by Hilbert transform of the signal 1.

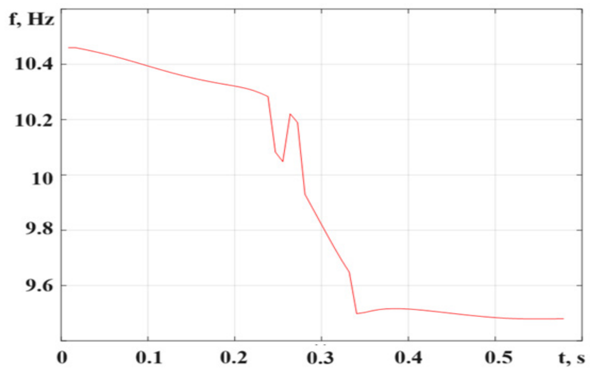

Figure 6. The instantaneous frequency dependence on time by Hilbert transform (signal 1)

The results obtained showed that Hilbert transform allows resolution of oscillations of close frequencies. However, it should be taken into account that Hilbert transform is used in the analysis of narrowband processes, while EEG fundamentally belongs to the class of broadband processes. That is why there is a certain discrepancy between the results obtained by the beating method and Hilbert Transform.

2.3. Zero-level Intersection Detector Method

The task of identifying hidden periodicities is an important task of spectral analysis, which allows one to determine the presence of a limited number of spectral components in a noisy signal. In practice, such a problem exists when solving data compression problems, when detecting resonance phenomena in technical devices, as well as in radars and telecommunications. If the number of detected periodic components of the signal is small, periodogram evaluation can be used to solve such a problem, but one should take into account the significant computational costs for its implementation. Thus, the search for fast and easy-to-implement methods for identifying periodicities in the original signal still remains relevant. This is explained by quite serious computing limitations even in modern microcontroller technology.

In this regard, the method of identifying periodicities based on zero crossings is important. This method is of interest due to low computational costs and simplicity of hardware implementation. If there is a dominant frequency in the signal spectrum, then it can be determined using zeros. However, this approach is effective only if there is only one periodicity in the signal and the signal-to-noise ratio is high.

Let us determine the average frequency using the zero-level intersection detector method (IDM). Despite of the slightly lower accuracy compared to the Hilbert transform, this method is attractive due to the possibility of easier practical implementation. Structure scheme of the corresponding device has been developed in [22]. According to the given method, the average frequency of frequency-modulated oscillation is approximately equal to [22], [27]:

$$f \simeq 1/2\Delta t \qquad (4)$$

where Δt is the time interval between two adjacent zero points of the frequency modulated signal.

Let n_0 is the number of zero points in the time interval T. Then the average time interval between two adjacent zero points is equal to $\Delta t = T/n_0$ and the previous equality can be written as

$$f \simeq 1/2T \qquad (5)$$

Figure 7 shows the instantaneous frequency dependence on time by zero-level intersection detector method for signal 1.

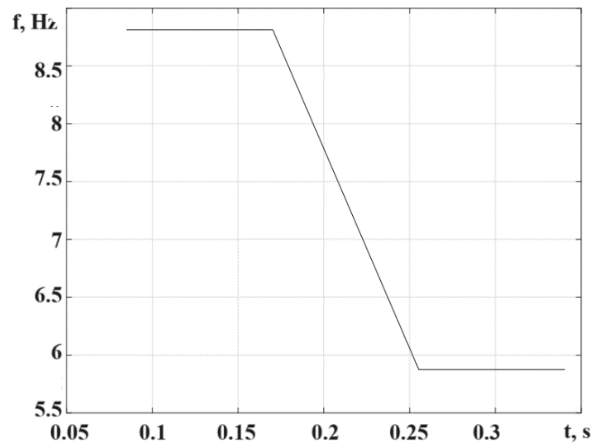

Figure 7. The instantaneous frequency dependence on time by zero-level intersection detector method (signal 1)

The results obtained by the zero-level intersection detector method are approximate because they are calculated by averaging several values. This explains their difference from the results obtained by the methods discussed above.

3. Results and Discussion

The results calculated on the basis of the considered methods are summarized in Table 1.

Table 1. Calculated results for α–rhythm frequency content

N of signal	1	2	3	4	5	6	7
f1(Hz) (BM)	9.79	9.28	8.84	9.13	8.91	8.8	9.31
f2(Hz) (BM)	10.2	9.76	9.44	9.6	9.25	9.4	9.87
f (FT) (Hz)	11.3	9.3 10.2	10.8 13.1	9.2 0.8	10.5 12.1	9.7	10.1
Frequency interval (HT)	9.5-10.5	8.9-9.5	8.9-9.5	8.7-9.6	8.7-9.3	7.9-9.8	8.7-9.9
Frequency interval (IDM)	5.9-8.9	5.9-9.9	5.9-9.8	6.9-9.7	6.9-9.4	6.7-9.9	6.9-9.4

FT is widely used in clinical practice. But EEG is a non-stationary signal and the use of Furrier transform is incorrect in this case [24]. This drawback is partly overcome by considering short implementations of the EEG [28], [29]. But this leads to a decrease in resolution. In addition, EEG diagnosis is currently limited to a very small number of neurological diseases [28], [29]. Therefore, the search for new methods for processing EEG opens up broad prospects in the development of diagnostic capabilities. Comparing the obtained data by the beating method and the Furrier transform showed that Furrier transform does not provide resolution of the nearest frequencies in all cases considered (Table 1). Detection of close frequencies can start a new stage in the phenomenological characteristic of the alpha rhythm and help in medical practice. This is especially important when processing short implementations that are used in screening diagnostics and mass examinations.

Hilbert transform allowed us to determine instantaneous frequencies. But this method also has limitations. It requires a narrowband signal and shows only the strongest frequency at a given time.

Data obtained from the Hilbert transform correlate with data obtained from the beating method (Table 1).

The difference may be partially explained by the fact that equal amplitudes of two close frequencies for the beating method.

Zero-level intersection detector method also allowed us to determine instantaneous frequencies. This method, despite its simplicity, has found wide application in radars [24]. It is less accurate than Hilbert transform because the frequency is averaged over several points. But comparing the data obtained based on Hilbert transform with the data obtained based on the zero-level intersection detector method, their correlation.

4. Conclusion

Researchers generally consider characteristics of the spindle such as: amplitude variability, duration, average amplitude, maximum peak frequency. But no one has noted that the spindle of the alpha rhythm is an additive mixture of other harmonics oscillations of close frequencies.

A new method for determining the information parameters of EEG is considered. Based on the beating method, the frequency content of the α–rhythm is calculated. A comparison of the results obtained with results based on Furrier transform showed that Furrier transform does not provide resolution of the nearest frequencies in all cases considered. It is shown that the use of the methods of analysis of non-stationary processes provides determination of the frequency content of the spindle and its dynamics. A fairly good match was obtained with the results based on the Hilbert transform and the beating method.

The results obtained using the zero-level intersection detector method differ quite significantly from the results obtained by the methods discussed above. This is explained by the fact that this method uses averaging of values.

One of the prospects of the beating method is the detection of close frequencies.The results obtained in the work can be recommended for implementation in medical practice. Only the use of a group of methods will provide a possibility to make a breakthrough in the diagnosis of EEG.

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