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Mathematics as Brain Training

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***Abstract* – The work focuses on the comparison of basic strategies in the teaching of mathematics. The first option is to use a calculator and ICT in order to solve more complex tasks and not waste time on algorithmically simple tasks. The second option is to solve all tasks with a paper-and-pencil strategy, while the number of tasks will be lower due to the fact that we need time for numerical calculation. Pupils of the 7th grade of elementary school were compared with pupils up to two grades higher in order to find out the differences between their performances. Pupils of the 7th grade do not use calculators during the math classes and thus have to cope with each task without help. Pupils of higher grades have been using calculators for numerically more challenging tasks for the last two years. Results of the nonmetric multidimensional scaling showed that the 7th grade pupils have the different position on the one-dimensional scale than the pupils of higher grades.**

***Keywords* – Mathematics, strategies, calculator, paper-and-pencil, solving tasks.**

1. Introduction

The teaching of mathematics, especially arithmetic in elementary schools takes place in the usual way through the recognition of numbers based on comparison.

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Pupils gradually master basic mathematical operations while the propaedeutic of equations is gradually created. As they progress through the grades, they expand their horizons regarding numerical sets and operations on them and gradually become familiar with new formulas.

In higher grades, the focus is on working with formulas and solving equations, calculators are used in the classroom so that pupils are not held back by algorithmic calculations (often with large numbers). Especially in the higher grades, it is assumed that the basic algorithms for calculations are already mastered at the required level and algebraization (working with expressions, formulas, and creating the necessary structures for solving tasks) is monitored.

1. Learning can also be seen as a form of training, there are two basic methods:To practice several tasks with the help of a calculator - substituting into formulas, editing expressions etc.
2. To practice just few tasks but with higher difficulty.

The first method is mainly focused on building mathematical intuition, which depends on the number of experiences [1]. Furthermore, the Resnick's work [2] also points to problems with intuition, and when it is necessary to disengage from intuition. The benefits of using the calculators in teaching mathematics are stated in a work [3], in which the authors point out the advantages in solving non-traditional tasks, tasks with extreme difficulty in analytical solution, and the basis for software solution of mathematical tasks is created. A better view of intuition and its need in problem solving can be found in the work [4], which refers to Simon Herbert's definition: "The situation will provide a stimulus, this stimulus allows access to information stored in memory, and the information will produce a response. Intuition is nothing more and nothing less than recognition". Subsequently, on the basis of this definition, he discusses what the access to information itself and the choice of the initial strategy for solving the problem depend on.

He points to a sequence of steps such as "System 1", which selects the information necessary to classify the problem, and "System 2", which analyses the proposed strategy, which it then rejects or advances as a possible solution.

The second method is a conservative approach, for which a good theoretical basis is presented in [5], where the authors focus on memory and its connection with the depth of text processing.

The experiment had two phases. In the first one, the subjects were shown a question that related to a specific word shown later. The questions were at different levels of text processing: from those not requiring in-depth processing of the given word (e. g. "Is there a word present?", "Is the word in capital letters?") to questions requiring a complex analysis of the given word (e. g. "Would the word fit in the following sentence?”). Subsequently, the word was displayed and subjects recorded their yes/no answer to a previously asked question. Then, the second phase followed and the subjects were given a sheet with the 40 original words and 40 similar distractors typed on it. Subjects were asked to check all the words they had seen in the first phase of the experiment.

The work [5] showed that the ability to remember words was significantly higher for those words that were covered by questions requiring in-depth analysis of the text.

2. Methods

Learning mathematics in a school environment is as a form of "training". In this sports terminology, it is also logical to assume that if someone has completed a longer course of study and is at a higher level, one will achieve a higher score in the tasks than someone who still has a lot to do.

The tasks selected from different areas for this study were not completely algorithmic and they were not a part of current curriculum for pupils included in the study. . The first group comprised the 8th and 9th grade pupils at the elementary school (denoted as Class 8.A, Class 9.A and Class 9.B). These pupils are moving from algorithmic solutions to tasks with main goals as algebraization, processing expressions, working with formulas, and solving linear equations and inequalities. Pupils use a calculator during regular lessons, so they do not waste time with classic paper-and-pencil calculations. Thanks to the use of a calculator, they are able to finish more tasks and to meet with different types of tasks and methods of solving them. They do not focus so much on the numerical calculations that the calculator will do for them, so they can focus more on building abstract thinking. The 7th grade pupils (Class 7.A) have mastered operations with natural numbers, positive decimal numbers and arithmetic operations with positive rational numbers. They even calculate application tasks in which complex numerical expressions and large numbers appear without using a calculator.

Tasks included in the study are described in section “Results” in detail:

1. Two independent tasks focused on the written addition algorithm (replacing letters with suitable numbers).
2. A task aimed at counting large numbers in triplets and comparing them with a specified value. The task also required a certain mathematical intuition in order not to waste time creating inappropriate triplets.
3. A task focused on a fraction as part of a whole. Dividing a whole into a small number of equal parts belongs to the propaedeutic of fractions and is a mostly intuitive part.
4. A task focused on the perimeter of a simple geometric shape.

These tasks were not computationally demanding (a large number of steps, many numbers, etc.) but focused on mathematical principles for individual tasks. For this reason, a higher overall score means a better understanding of the mathematical principles on which individual tasks are based.

The pupils’ results were analyzed by the Kruskal-Wallis rank test [6] to confirm that they are from the same distribution. Supposing that individual grades (or classes) can be somehow ordered on a one-dimensional scale according to the pupils’ results in the test, the ordinal multidimensional scaling (MDS) [7] procedures for one-dimensional case was used to uncover this hidden configuration of classes. Statistical analyses and visualizations were performed using R statistical software [8] and packages circlize [9], smacof [7], [10], and vegan [11].

3. Results

A pilot study consisted of the following 5 tasks:

1. Replace letters with numbers, so that the addition holds true:

$$\begin{array}{c} \mathrm{O\,K\,O} \\ \mathrm{O\,K\,O} \\ \hline \mathbf{K\,U\,K} \end{array}$$

2. Replace letters with numbers, so that the addition holds true:

$$\begin{array}{c} \mathrm{B\,A\,M} \\ \mathrm{B\,D\,D} \\ \hline \mathbf{M\,D\,A} \end{array}$$

3. A car with a carrying capacity of 5 tons has to transport all of the boxes up to three times. Find a way to do it, if boxes' weights are the following:

2380 kg, 2050 kg, 1120 kg, 1100 kg, 560 kg, 950 kg, 3230 kg, 70 kg and 2550 kg.

4. There were 18 candies in the chocolate box. Robert took one third of them. Norbert also took one third of them. How many candies are left in the box?

5. The lifeguard walked once around the rectangular pool. He thus walked 140 m. One side of the pool is 50 m long. What is the length of the other side?

a) 20 m
b) 40 m
c) 90 m
d) 100 m

All tasks were evaluated on a binary scale (correct or wrong answer), Tables 1 and 2 summarize the pupils' results.

Table 1. Number of tasks solved by pupils in each class

	0	1	2	3	4	5	Number of pupils
Class 7.A	2	1	1	2	3	1	10
Class 8.A	1	5	4	5	0	0	15
Class 9.A	3	4	2	1	0	1	11
Class 9.B	1	4	1	3	1	1	11

Table 2. Number of pupils who solved individual tasks

	Task 1	Task 2	Task 3	Task 4	Task 5
Class 7.A	7	3	6	4	6
Class 8.A	1	0	6	10	11
Class 9.A	1	1	5	4	5
Class 9.B	3	1	8	5	7

a. Class comparison results

Figure 1 shows the distribution of the total score (from Table 1) in each class.

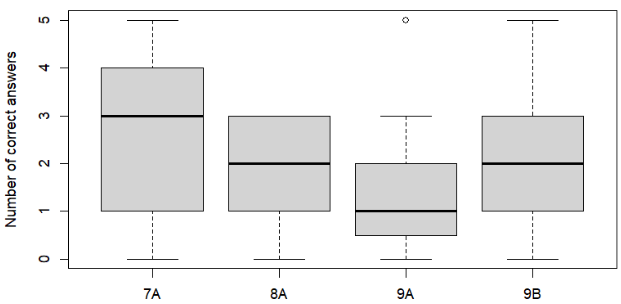

Figure 1. Boxplots of total score for individual classes

To see if the pupils' results in all 4 classes came from the same distribution, the nonparametric Kruskal-Wallis rank test [6] was used. The hypothesis of the same underlying distribution cannot be rejected on the 5 % significance level (p-value 0.308).

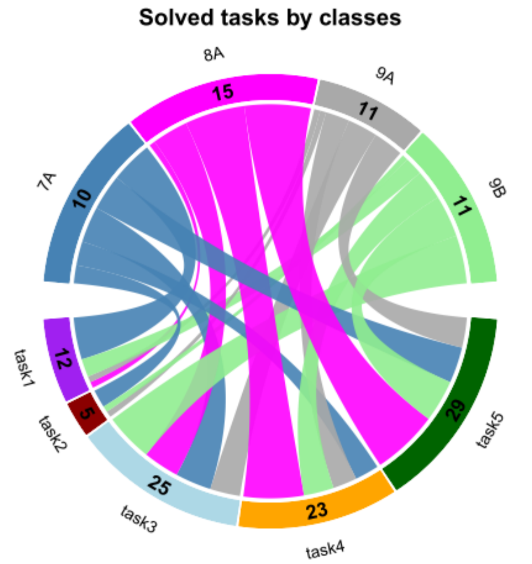

Figure 2. Visualization of solved tasks in classes

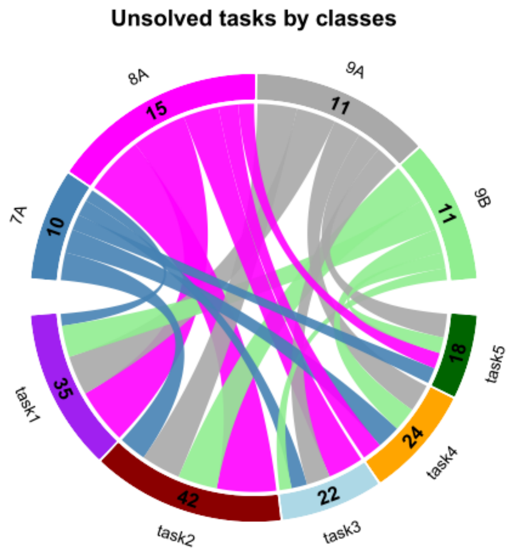

Figure 3. Visualization of unsolved tasks in classes

From Figure 2 and Figure 3 can be seen that the most (29) of all 47 pupils solved the task 5 and the less of them (5) solved the task 2. None of the pupils from the Class 8.A solved the task 2.
There is also the light difference between the successes of solving the tasks among the 7th grade pupils and among the 8th and 9th grade pupils. Approximately the same number of 7th grade pupils solved each task. But among the 8th and 9th grade pupils, tasks 1 and 2 were solved by smaller number of pupils, whereas the greater number of pupils solved the tasks 3, 4 and 5.

b. Results of multidimensional scaling

Supposing that 4 classes can be somehow ordered according to the pupils' results in the test and assuming one dimension (as a measuring scale of pupils' calculation ability or performance in the test, for example), the task is to find the hidden configuration of classes on the real line. According to the binary evaluated tasks, the ordinal multidimensional scaling (MDS) [7] procedures with results in only one dimension was used to uncover the resulting configuration of classes. The input data was the matrix of 11 variables and 4 classes (Table 3). Pearson correlation as a similarity measure between classes was used and after fitting the MDS models the goodness-of-fit assessment of each model was performed. Figure 4 shows the resulting configuration for the first model of ordinal MDS with stress value 0.002 (stress values < 0.2 are acceptable, values < 0.05 indicate very good models).

Table 3. Input data matrix for multidimensional scaling

	Total score 0	Total score 1	Total score 2	Total score 3	Total score 4	Total score 5	Task 1 solved	Task 2 solved	Task 3 solved	Task 4 solved	Task 5 solved
Class 7.A	0.200	0.100	0.100	0.200	0.300	0.100	0.700	0.300	0.600	0.400	0.600
Class 8.A	0.067	0.333	0.267	0.333	0.000	0.000	0.067	0.000	0.400	0.667	0.733
Class 9.A	0.273	0.364	0.182	0.091	0.000	0.091	0.091	0.091	0.455	0.364	0.455
Class 9.B	0.091	0.364	0.091	0.273	0.091	0.091	0.273	0.091	0.727	0.455	0.636

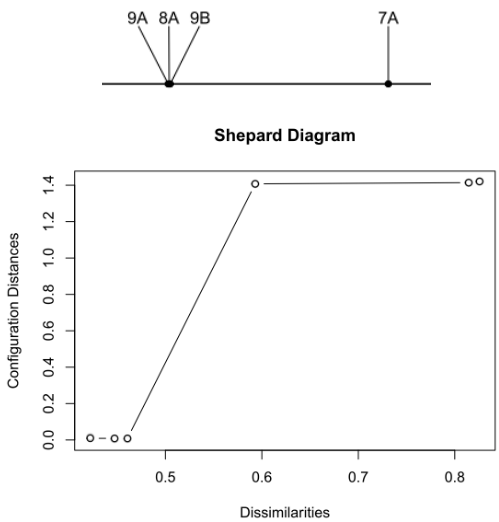

Figure 4. Result of ordinal MDS (up) and its Shepard's diagram (down)

Class 7.A is projected away from other three classes; these classes are projected into the same point on the measuring scale. The bootstrap validation of this configuration by SMACOF Bootstrap (bootmds from smacof R-package) with 500 replications leads to mean bootstrap stress 0.011 with 95 % confidence interval (0, 0.159) and stability coefficient 0.844. That confirms the very good quality of this model.

The same configuration resulted also from the nonmetric multidimensional scaling with stable solution from random starts (metaMDF from vegan R-package, [11], see Figure 5. Stress value is 7.96e-05 and test statistics for goodness of fit for each point in the configuration are 0, 3.49e-05, 5.52e-05 and 4.56e-05 (large values indicate poor fit).

From both models, we conclude that pupils from Class 7.A have a different position on a one-dimensional measuring scale according to their calculation performance in the test than pupils of the other 3 classes 8.A, 9.A and 9.B. That confirms the assumption, that pupils of the lowest grade (Class 7.A) mastered the tasks different way than pupils of the higher grades.

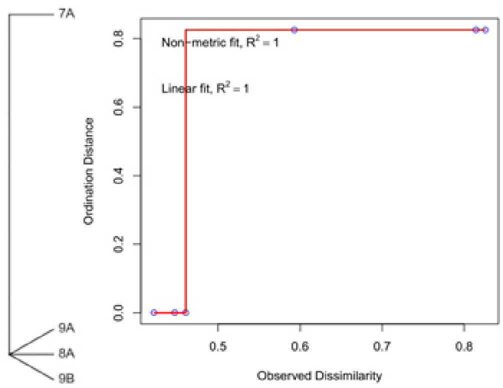

Figure 5. Result of nonmetric MDS (left) and its Shepard´s diagram (right)

4. Conclusion

Given that tasks did not have a high computational complexity, but were focused on understanding the mathematical essence related to the problem, it is appropriate to expect a better success rate for the higher grades. Despite this, it turns out that pupils who learn without a calculator have different calculation abilities (by inspecting their solutions of the tasks detailed). As long as the task is not purely algorithmic, the successful solution depends on the choice of a suitable strategy, which means that mathematical intuition is of great importance. Thinking about intuition based on Herbert, memorizing the appropriate strategies is crucial, because it is impossible to choose the most suitable strategy for solving the task without knowing the large set of strategies. In this step it turns out that a higher rate of memorization depends more on the depth of processing than on repetition. Another dominant part of the learning process is the time dependence to information processing [12]. That means it is not only important to show the information and work with it, but there is also a time needed to absorb the information provided. Using the calculator speeds up the process of solving pupils' tasks. On the other hand it can take away the necessary time for information processing.

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