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# Polar Response of a Circular Piston

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***Abstract* –The current paper reviews theoretically and examinesexperimentally the polar plotof a loudspeakerin the far and near field.An original equation for sound pressure calculation in the near and far field is proposed. An experiment is conducted to confirm the theoretical data and the accuracy of the proposed expression. Several conclusions are drawn from the analysis of the study.**

***Keywords* –polar plot, near field,far field, sound pressure, measurements.**

## 1. Introduction

The controlling and modeling of loudspeakers' and microphones' radiation pattern has become a separate scientific branch of acoustics. Through deeper understanding of the problem'snaturethe examination and improvement of microphones' radiation has become possible[1],as well as the manufacturing of directed [2]andhyper directedloudspeakers[3].When directed sound pressure is generated by powerful sound waves, one can overcome the gravitational force[4]or exciteliquid, during which process, short bursts of light are emitted[5]. That process is called sonoluminescence. During sonoluminescence sound wavesform little bubbles in the liquid, that bloat and implode quickly, which results in producing light. At the same time huge amount of heat is released.

Directed microphones and loudspeakers are often used in architectural acoustics as well[6].

In the present paper the radiation pattern of a circular piston in the near [7] and far [8] field is theoretically reviewed and experimentally examined.

## 2. Theoretical background

The object of this study is a circular piston. Due to its symmetry with respect to the *z* axis (Fig.1) one can assume that its polar pattern will be one with rotational symmetry. To define that symmetry it is sufficient to determine the sound pressure level at distance $r$ in a plane whose normal is perpendicular to the $z$ axis.

The sound pressure created by the emitter in the environment can be calculated under the assumption that its surface consists of multiple elementary sections $dQ$.

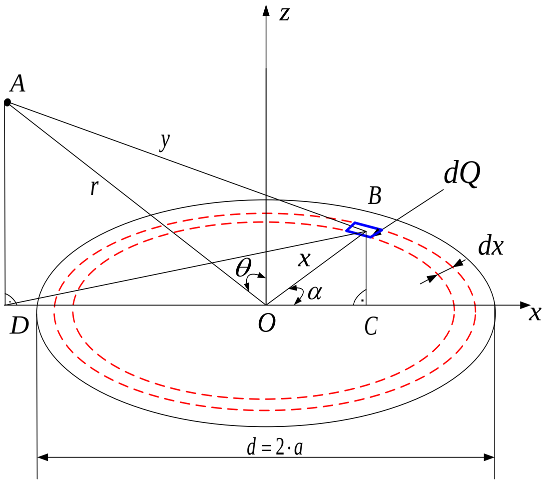


*Figure 1. Geometry of a circular piston directivity* [9].

The surface area of each elementary section can be determined using the following relation[10]:

$$dQ = x \cdot dx \cdot d\alpha \,. \qquad (1)$$

Each of the elementary section creates sound pressure in point *A*. Point *A*is situated at distance *y*and angle $\theta$ with respect to the $z$ axis. The sound pressure is[11]:

$$dp_{\theta} = \frac{r_0 p_m}{y} e^{j(\omega t - ky)}, \qquad (2)$$

where: *y*is the distance between the elementary sectionand point*A* (Fig. 1);

$p_m$ – theinstantaneous amplitude of the sound pressure;

$r_0$ –the radius of the point section.

With reference to Fig. 1 where angle DCB = 90º one can derive several expressions.

$$\overline{DC} = \overline{DO} + \overline{OC} = r\sin\theta + x\cos\alpha \,; \qquad (3)$$

$$\overline{BD}^2 = \overline{DC}^2 + \overline{CB}^2 = \\ = (r\sin\theta + x\cos\alpha)^2 + x^2\sin^2\alpha = \\ = r^2\sin^2\theta + 2rx\sin\theta\cos\alpha + x^2 \quad (4)$$

$$\overline{AB}^2 = \overline{BD}^2 + \overline{AD}^2 = \\ = r^2\sin^2\theta + 2rx\sin\theta\cos\alpha + x^2 + r^2\cos^2\theta \quad (5)$$

Equation (5) can also be represented as:

$$y^2 = r^2 + x^2 + 2rx\sin\theta\cos\alpha \quad (6)$$

or:

$$y = \sqrt{r^2 + x^2 + 2rx\sin\theta\cos\alpha}\ . \quad (7)$$

Instantaneous amplitude of the sound pressure is [12]:

$$p_m = \rho_s c_0 k r_0 v_m . \quad (8)$$

where : $c_0$ is aspeed of sound;

$k = 2\pi/\lambda$ – wave number;

$v_m$ – particles velocity;

$\rho_s$ – density of the environment;

$\lambda$ – wave length;

After multiplying both sides of the expression (8)by $r_0$, and also multiplying the right hand side by $2\pi/2\pi$ one can take into consideration that the surface of the point emitter is $2\pi r_0^2 = dQ$ and subsequently:

$$r_0 p_m = \frac{\rho_s c_0 k v_m}{2\pi} dQ . \quad (9)$$

If expressions (1) and (9) are substituted in equation (2), the elementary sound pressure in the direction defined by the $\theta$ angle will be as follows:

$$dp_\theta = \frac{\rho_s c_0 k v_m}{2\pi y} e^{j(\omega t - ky)} x dx d\alpha . \quad (10)$$

After expression (7) is substitutedin the previous equation (10) and it is afterwards applied to the entire surface of the piston, the following original equation for calculating the total sound pressure level, created by all point emitters, can be rewritten:

$$p_\theta = \frac{\rho_s c_0 k v_m}{2\pi} e^{j\omega t} \int_0^a x dx \int_0^{2\pi} \frac{e^{-jk\left(\sqrt{r^2+x^2+2rx\sin\theta\cos\alpha}\right)}}{\sqrt{r^2+x^2+2rx\sin\theta\cos\alpha}} d\alpha . (11)$$

For the far field, when the distance is significantly larger than the diameter of the piston ( $r >> d$ ),the denominator of the integrantis $\sqrt{r^2 + x^2 + 2rx\sin\theta\cos\alpha} = r$ (Fig. 2), and the numerator is $e^{-jk\left(\sqrt{r^2+x^2+2rx\sin\theta\cos\alpha}\right)} = e^{-jk(r + x\sin\theta\cos\alpha)}$ (Fig. 3).

After the new denominator and numerator are replaced in expression (11),the well-known expression for calculating sound pressure level, created by all point emitters, can be rewritten [11]:

$$p_\theta = \frac{\rho_s c_0 k v_m}{2\pi r} e^{j\omega t} \int_0^a x dx \int_0^{2\pi} e^{-jk(r + x\sin\theta\cos\alpha)} d\alpha . (12)$$

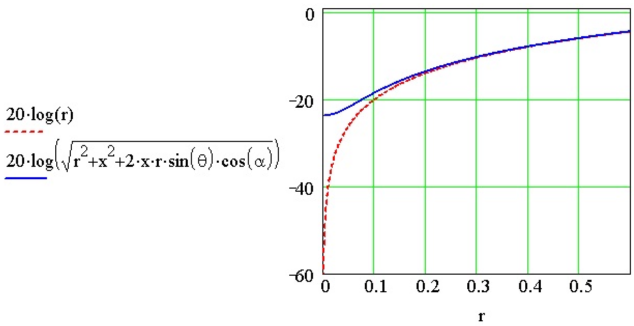


*Figure 2. Graphic representation of the denominators in the newly-proposed original expression (11) – solid blue lineand the well-known expression(12) –dashed red line.*

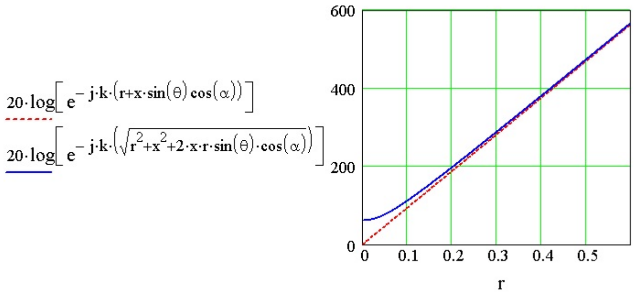


*Figure 3. Graphic representation of the numerators in the newly-proposed original expression (11) – solid blue line and the well-known expression (12) –dashed red line.*

After integrating by$\alpha$,expression (12)becomes[13]:

$$p_\theta = \frac{\rho_s c_0 k v_m}{r} e^{-jkr} \int_0^a J_0(kx\sin\theta) x dx ,(13)$$

where $J_0(kx\sin\theta)$ is Bessel function of the zero kind of the argument $(kx\sin\theta)$, and after integrating by $x$, one obtains[11]:

$$p_\theta = \frac{\rho_s c_0 k v_m a^2}{r} e^{-jkr} \frac{J_1(ka\sin\theta)}{ka\sin\theta} , \quad (14)$$

where $J_1(ka\sin\theta)$ is Bessel function of the first kind of the argument $(ka\sin\theta)$ .

The Bessel function can also be represented as[14]:

$$J_1(ka\sin\theta) = \\ = \frac{ka\sin\theta}{2} - \frac{k^3a^3\sin^3\theta}{2^24} + \frac{k^5a^5\sin^5\theta}{2^24^26} - ..., \quad (15)$$

thereforethe interrelation after the $e^{-jkr}$ in expression (14) can berewritten[9]:

$$\frac{J_1(ka\sin\theta)}{ka\sin\theta} = \\ = \frac{1}{2} - \frac{k^2a^2\sin^2\theta}{2^24} + \frac{k^4a^4\sin^4\theta}{2^24^26} - ... . \quad (16)$$

The radiation pattern*G(θ)*is derived from the interrelation[11]:

$$G(\theta) = \frac{p_\theta}{p_{0^\circ}}, \quad (17)$$

where $p_{0^\circ}$ isthe amplitude of the sound pressure level at the acoustic axis.

To obtain a radiation pattern it is necessary to determine the sound pressure levelfor $\theta = 0^\circ$ . Due to the zero value of the $\theta$ angle, the expression (16) becomes:

$$\frac{J_1(ka\sin 0^\circ)}{ka\sin 0^\circ} = \frac{1}{2}, \quad (18)$$

and for the sound pressure level in that direction the following is valid:

$$p_{0^\circ} = \frac{\rho_s c_0 k v_m a^2}{r} e^{-jkr}\frac{1}{2}. \quad (19)$$

If expressions (14) and (19) are substituted in equation (17), the polar response of a circular piston in the far field will be:

$$G(\theta) = 2\frac{J_1(ka\sin\theta)}{ka\sin\theta}. \quad (20)$$

There are zeros to be expected in the radiation pattern, when $k \cdot a \cdot \sin\theta$ is a root of Bessel function: 3.83 – Fig. 4; 7.02 – Fig. 5; 10.2 – Fig. 6; 13.3 – Fig. 7; 16.5.

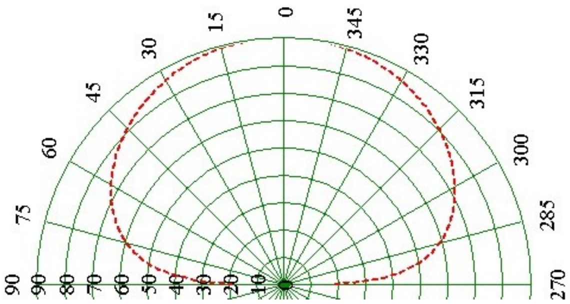

*Figure 4. Polar responseat:*
$k \cdot a \cdot \sin\theta = 3.83$.

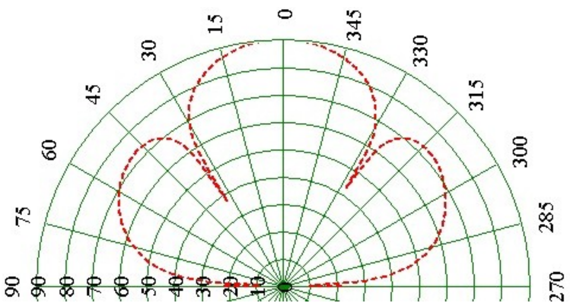

*Figure 5. Polar responseat:*
$k \cdot a \cdot \sin\theta = 7.02$.

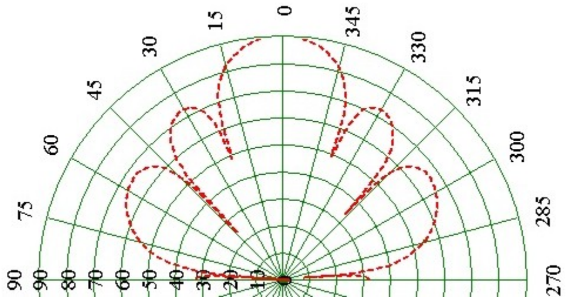

*Figure 6. Polar responseat:*
$k \cdot a \cdot \sin\theta = 10.2$.

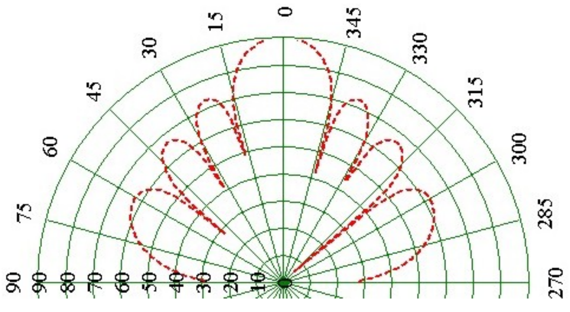

*Figure 7. Polar responseat:*
$k \cdot a \cdot \sin\theta = 13.3$.

In Fig. 8 ÷ 10comparison is made between the newly-proposed original expression (11) and the well-known expression (12) for different distances and *k·a·*sin*θ*= 7.02.

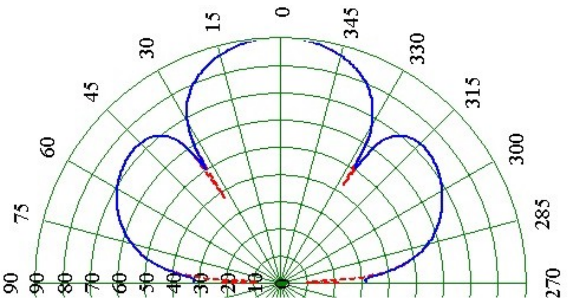


*Figure 8. Polar responseat 2.40 m and* $k \cdot a \cdot \sin\theta = 7.02$:
*Solid blue line – newly-proposed original expression (11);*
*Dashed red line – well-known expression (12).*

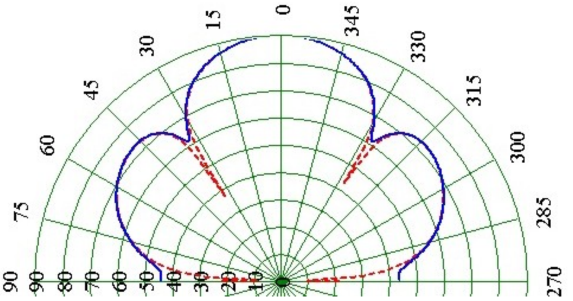


*Figure 9. Polar responseat0.60 mand* $k \cdot a \cdot \sin\theta = 7.02$:
*Solid blue line – newly-proposed original expression (11);*
*Dashed red line – well-known expression (12).*

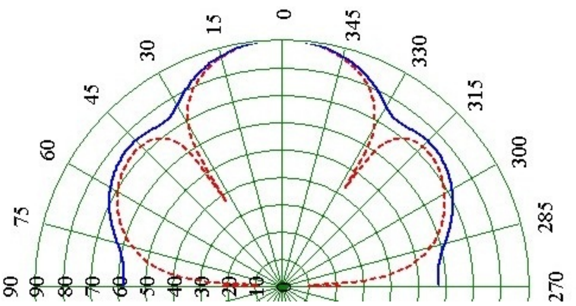


*Figure 10. Polar responseat 0.15m and* $k \cdot a \cdot \sin\theta = 7.02$:
*Solid blue line – newly-proposed original expression (11);*
*Dashed red line – well-known expression (12).*

## 3. Experimentalsetup and results

The practical measurements werecarried outwith a sound-level meter Robotron Präzisions,a microphoneMK301, a sound card Realtek High Definition Audio and aMatlab®program [15]using a time selection method[16].The object of the study is a JBL loudspeakerwith nominal diameter of 0.13 m. The parameters were selected so that $k \cdot a \cdot \sin\theta = 7.02$. The measurements took place in an anechoic chamber at Technical University – Varna.

Results from the measurements at different distances– 2.40 m, 0.60 mand 0.15 mfrom the loudspeakerare visually represented in Fig.11 ÷ 13.

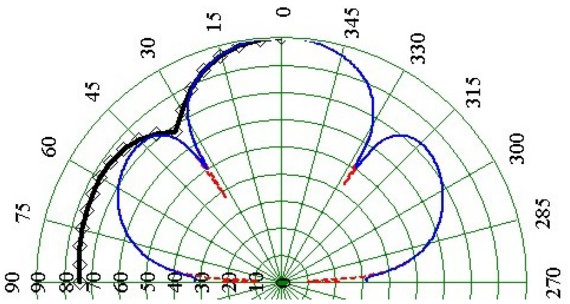


*Figure 11. Polar responseat 2.40 m from the loudspeaker and* $k \cdot a \cdot \sin\theta = 7.02$:
*Solid blue line – newly-proposed original expression (11);*
*Dashed red line – well-known expression (12);*
*Solid black line with diamonds – experimental data (0º ÷ 90º).*

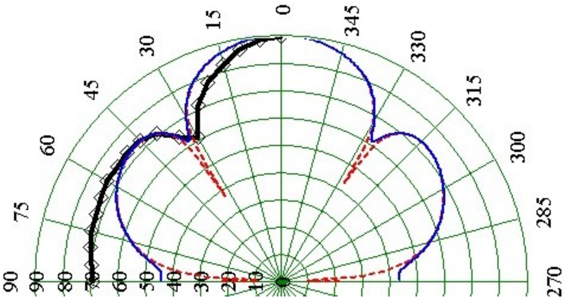


*Figure 12. Polar responseat 0.60 m from the loudspeaker and* $k \cdot a \cdot \sin\theta = 7.02$:
*Solid blue line – newly-proposed original expression (11);*
*Dashed red line – well-known expression (12);*
*Solid black line with diamonds – experimental data (0º ÷ 90º).*

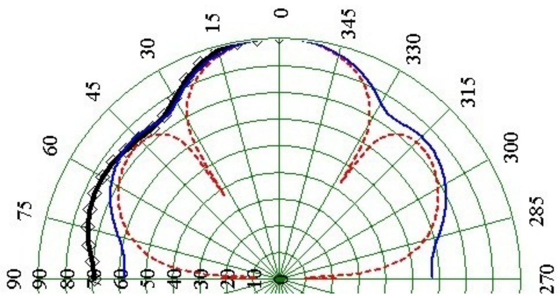


*Figure 13. Polar responseat 0.15 m from the loudspeaker and* $k \cdot a \cdot \sin\theta = 7.02$:
*Solid blue line – newly-proposed original expression (11);*
*Dashed red line – well-known expression (12);*
*Solid black line with diamonds – experimental data (0º ÷ 90º).*

## 4. Conclusion

A study of the well-known expression (20) for far field polarresponse calculation reveals several conclusions:

- the polar pattern form depends on the relation between the loudspeaker's diameter (respectively radius*a*)andthe emitted sound wave's length (wave number*k*);
- when the size of the loudspeaker is significantly smaller than the emitted

wave length, then the $k \cdot a \cdot \sin\theta \approx 0$ and $G(\theta) \approx 1$.

From the experimental data analysis of the expressions (11) and (12) and their graphic representation (Fig.11÷13),one can deduce:

- the newly-proposed, by the author generalized expression for sound pressure level calculation (11) is more accurate compared to the well-known expression (12)in regards to the near field and hence it should be used for determining the sound pressure level in it;
- practically, in the space directly in front of the loudspeaker the radiation pattern lacks pronounced minimums, while in the far field the minimums become commeasurable with the measuring system's (microphone, sound card) proper noises;
- the greater the distance the more the graphic representation of the two expressions – the one proposed by the author (11) and the well-known (12) – overlapses and in the far field it becomes completely identical.

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