Delaying Scaffolding Using GeoGebra: Improving the Ability of Vocational Students to Draw Conclusions

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Abstract **– This case study investigated the implementation of the delayed scaffolding strategy by the teacher using GeoGebra software in learning the topic of absolute functions. This strategy aimed to assist the vocational students to understand the representation of the expression of absolute function symbols in graphical form. The results of the study show that the implementation of delayed scaffolding using GeoGebra improved students' ability to draw conclusions in understanding the representation of absolute functions in graphical form. This strategy has positive effects on students in the process of learning mathematics, especially for vocational students in Indonesia.**

Keywords **– Delayed scaffolding, GeoGebra, vocational students, draw conclusions, absolute functions.**

1. Introduction

Mathematics teaching cannot be separated from the scaffolding component. In the learning process in the classroom, scaffolding means assistance from competent people to students if they need help in achieving learning goals [1]. Various studies show that scaffolding is effective in learning mathematics [2],[3] and developing students' critical thinking skills [4],[5]. Critical thinking ability is needed by the students to face real-world problems [6].

Various scaffolding tools are used in the learning process, including the use of technology [7],[8]. Technology is essential in the process of learning mathematics, especially for vocational students. The vocational school in Indonesia is secondary education that prepares students to work in certain fields [9]. The aim of vocational schools is to develop graduates with certain skills to be competent in their work and to develop themselves. To become a competent graduate, one of the skills that vocational students must possess is the ability to think critically. The critical thinking ability is an essential requirement in the current technological era [10], which includes the ability to make conclusions [11]. This demands the vocational teachers to facilitate students to obtain the best learning outcomes.

The vocational school curriculum is different than that of a high school. It requires more practice lessons than theoretical lessons. Mathematics is one of the fundamental theoretical lessons in the vocational school curriculum. However, time allocation to learn mathematics in vocational schools is less than in high school. The need to improve quality of human resources in vocational education [12] and the limited allocation of theoretical teaching hours is a challenge for teachers in the process of learning mathematics, including the topics of math functions, such as the absolute function. The students often struggle to understand the translation of algebraic representations between symbols and graphs of absolute functions, even though understanding the translation of mathematical representations is essential in understanding the concept of a function [13]. Various studies show student difficulties in understanding the translation of

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mathematical representations [13],[14],[15]. To overcome student difficulties in the translation of function representation, appropriate learning media are needed.

Scaffolding using technology has benefits in the learning process. Teachers play an important role in regulating how to integrate students and technology in supporting mathematical thinking [16]. The combination of tools and scaffolding goals is called a scaffolding strategy [17]. The use of media needs to be considered by the teachers to support students in learning mathematics. It can include computer-based media, such as the GeoGebra, which is software for learning mathematics, specifically geometry and algebra [18]. Various studies show that the use of GeoGebra has a positive effect on learning mathematics [19],[20].

However, studies that explore the effectiveness of scaffolding implementation in the classroom are limited [21]. This study is based on the practice of teacher scaffolding in mathematics class, namely delayed scaffolding, which is a time management strategy in which the teacher does not immediately give scaffolding to students. Delayed scaffolding has a positive effect on student learning outcomes [22],[23]. The low number of studies that present the implementation of delayed scaffolding in learning mathematics in the classroom encouraged this study to further explore the delayed scaffolding strategy. The case study aimed to investigate and describe the actions of teachers to postpone scaffolding to help students understand graph representations of absolute functions using GeoGebra. The effect of the delayed scaffolding method using GeoGebra is discussed below.

2. Methods

This was a qualitative study with a descriptive case study approach. A descriptive case study is a way to present a complete description of a phenomenon in the studied context [24], namely the teacher's scaffolding strategy. The scaffolding strategy in this study is the action of the teacher to delay scaffolding using GeoGebra in the topic of absolute function graphs. GeoGebra was used by teachers to help students understand the absolute function of algebraic representations in graphical form. The subject was a teacher at one of the vocational schools in Malang, Indonesia. The school was selected because, in the classroom, every student can learn mathematics using a laptop with the GeoGebra software. All students had the opportunity to use GeoGebra when the learning process took place. The teacher was selected because he met the predetermined criteria, namely teachers with knowledge of the absolute function topic who are skilled in using the GeoGebra software and can provide good information on the reasons for implementing delayed scaffolding that they have done in class.

Data used in this study were observations, video recordings, students' work, and interview transcripts. Observations were made by recording the teacher's actions when delaying scaffolding during the learning process in the class majoring in computer networks. The absolute function was taught in two meetings in class, with one meeting lasting for 90 minutes. The interview was conducted after the observation activity by showing the results of the video learning that had been done. The video of the recorded learning was shown to the teacher to remind him of the moment of delayed scaffolding he had done. The interview aimed to explore in-depth information of the teacher's interesting and important actions during the observation process. Data were analysed to obtain process descriptions and the effect of learning strategies carried out by the teacher.

3. Results and discussion

The learning process began by teaching students to recognise the algebraic representation of the absolute function graph using the method of determining the sample points passed by the graph using tables. This method was done by the teacher to remind students of the method of drawing graphics at the junior high school level. Students were expected to recognise the characteristics of the absolute function graph, such as the letter "v", the symmetry line, and that it is located above the x-axis. The learning time of absolute functions was very limited, which does not allow students to explore the nature of absolute functions using tables. To help the students understand the nature of absolute functions properly, the teacher guided them using GeoGebra to draw graphs of functions.

Learning by understanding the properties of absolute functions was done using the graph posing method. This method presents a number of graphical functions in a structured and sequential manner to the students. The learning outcome of this topic was that the students could explain the influence of each element $a, b,$ and c in the absolute function of $f(x) = a|x + b| + c$. To achieve the learning objective, the teacher divided learning activities into three stages. In the first stage, the teacher first guided students to draw a graph of the $f(x) = |x|$ function and to match it with the results of the graphs that the students made manually. Then, the graph was drawn using GeoGebra. The teacher assigned students to draw a graph of the functions $f(x) = \frac{1}{3}|x|$ and $f(x) = 3|x|$ with the aim that students understand the effect of changing the values of a. The teacher then asked the students to observe the graph (Figure 1.).

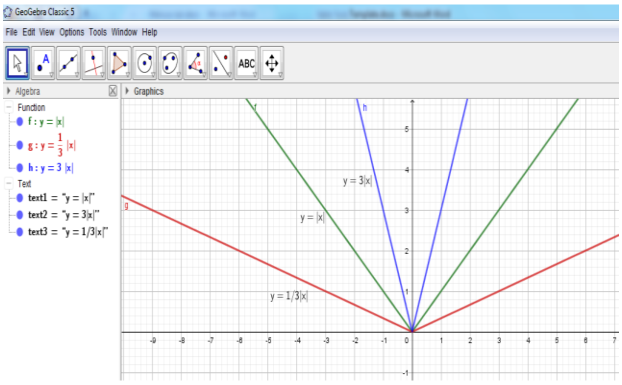
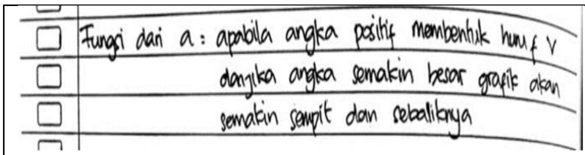


Figure 1. Graphs of $f(x) = |x|$; $g(x) = \frac{1}{3}|x|$, *and* $h(x) = 3|x|$

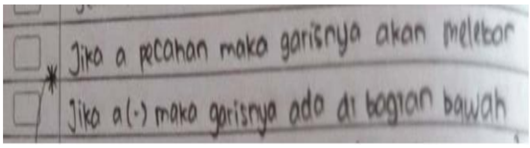
After the students observed the graphs, the teacher asked them to make conclusions regarding the effect of changing the value using their own words. The teacher asked the students to write conclusions based on observing the graphs of the three functions. Figure 2. are the examples of students' work.



Fungsi dari a : apabila angka positif membentuk huruf V
dan jika angka semakin besar grafik akan
semakin sempit dan sebaliknya

(a)

Translated (a):
The function of a: if a is positive, then the graph forms the letter V. The larger the value of, the narrower the graph, and vice versa.



Jika a pecahan maka garisnya akan melebar
Jika a (-) maka garisnya ada di bagian bawah

(b)

Translated (b):
If a is a fraction, then the graph is wider.
If a is negative (-), then the line is at the bottom

Figure 2. Examples of translated students conclusion of the effect of changing the value of a

In the second assignment, the teacher asked the students to draw a graph of $f(x) = |x|$; $f(x) = |x + 3|$; and $f(x) = |x - 3|$. These functions were chosen by the teacher with the aim that the students understand the effect of changing the value of b on the generated graph (Figure 3.).

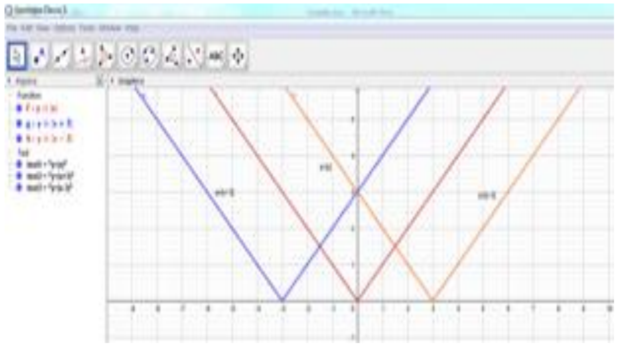
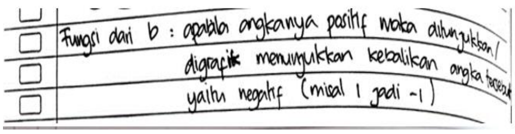


Figure 3. Graphs of $f(x) = |x|$; $g(x) = |x + 3|$; *and* $h(x) = |x - 3|$

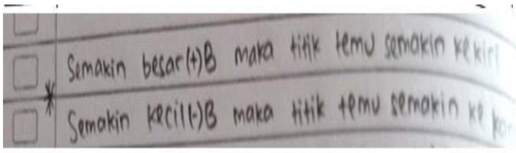
Next, the teacher asked the students to write conclusions of the generated graphs using their own words. The student conclusions are presented in Figure 4. From the teacher's observations of the conclusions made by the students, there were inaccurate conclusions, as shown in Figures 2. and 4. The students' writing describes their thinking and can be used by teachers as a tool to assess their understanding.



Fungsi dari b : apabila angkanya positif maka ditunjukkan/
digrafik menunjukkan kebalikan angka tersebut
yaitu negatif (misal 1 jadi -1)

(a)

Translated (a):
The function of b: if it is positive, then the graph shows the inverse (negative) of the value (e.g. 1 converted to -1)



Semakin besar (+) B maka titik temu semakin ke kiri
Semakin kecil (-) B maka titik temu semakin ke

(b)

Translated (b):
As the B (+) increases, then the intersecting point moves to the left.
As the B (-) decreases, then the intersecting point moves to the right

Figure 4. Examples of translated students' conclusion of the effect of changing the value of b

The next learning stage was to discuss the conclusions that had been made. Even though the teacher knew of the students' mistakes, he did not immediately give the correct conclusions to the students. At this moment, the teacher implemented the delayed scaffolding strategy. The delayed scaffolding strategy is metacognitive scaffolding [25] that allows students to be directly involved in the process of discovering the meaning of changing a value in an absolute function. The teacher used scaffolding tools in the form of questions to stimulate the students to produce information [26] in making the right conclusions with the help of GeoGebra that presents visual representations which strongly support the learning process [27]. Effective representation is the basis of mathematical reasoning [28]. The transcribed conversation between the teacher (T) and the students (S) when discussing the graphs in Figure 3. is as follows.

T : What is the difference between the three graphs?
S : The intersection point is different
T : How are they different?
S : The first graph has the intersection point in the middle, the second one is on the left, and the third one is on the right.
T : So what can we conclude of the effect of the value of b on the graphs? Write your conclusion in your book.

The questions asked by the teacher focused on the differences in the three function graphs that had been presented. With the observation of the chart patterns, students were expected to analyse and make conclusions on the effect of changing the value of a in the function of $f(x) = a|x + b| + c$ (Figure 1.). Multiplication of $f(x) = |x|$ with a changed the width of the graph of $f(x) = |x|$. If $|a| < 1$, the graph widened, while in contrast if $|a| > 1$, the graph narrowed. As can be seen in Figure 3., the change in the value of b shifted the graph horizontally. If $b > 0$, $f(x)$ was translated by b units to the left to obtain $g(x)$. If $b < 0$, $f(x)$ was translated by b units to the right to obtain $h(x)$. Effective teacher questions can bring positive learning outcomes for students [29].

To teach students regarding the effect of the constant c on the absolute function of $f(x) = a|x + b| + c$, the teacher took a slightly different approach to the two previous activities. In this activity, the teacher displayed a graph of absolute functions in stages, which involved the students' initial knowledge regarding the effect of changes in the values of a and b. The teacher presented graphs sequentially from $f(x) = |x|, f(x) = |x + 3|$, and $f(x) = 2|x + 3|$. Next, students were assigned to draw the function of $f(x) = 2|x + 3| - 4$, and they were asked to observe the differences from the previous graphs. Then, the teacher assigned students to draw a graph of the function $f(x) = 2|x + 3| - 4$ and again asked students to look at the generated graph. Lastly, the teacher asked students to observe the graphs of $f(x) = 2|x + 3|, g(x) = 2|x + 3| + 4$, and $h(x) = 2|x + 3| - 4$ (Figure 5.). The students' writing can be used by the teacher to understand their learning progress in class [30].

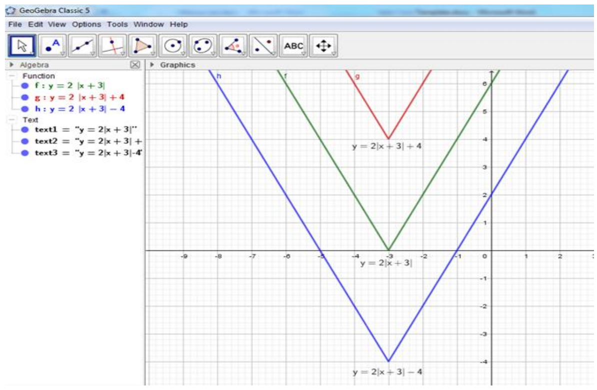
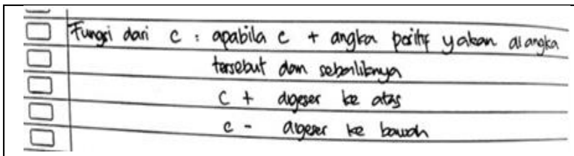


Figure 5. Graphs of $f(x) = 2|x + 3|, g(x) = 2|x + 3| + 4$, and $h(x) = 2|x + 3| - 4$

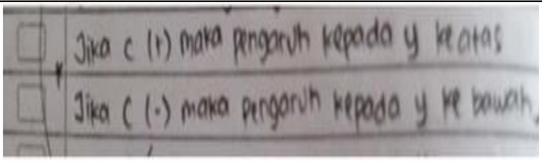
After students observed the graphs, the teacher assigned students to write conclusions about the effect of the value of c on the graph of $f(x) = a|x + b| + c$. Similar to the previous activities, the teacher again asked students about the differences between the three function graphs that had been presented. The teacher also asked students to explain the effect of changes in the values of a, b, and c to strengthen the understanding of the learned topic. To determine students' understanding of the shape of the absolute function graph, the teacher assigned students to express the absolute function graph verbally without drawing it. Learning by repeating the previously learned items will be stored in long-term memory [31].

In the third activity, when students made conclusions using their own words, they spent more time than the previous two activities. Students needed time to analyse the effect of changes in the three values of a, b, and c. This longer time was one effect of previous learning experiences that made students more careful in drawing conclusions from their observations. There is a cognitive bias that will influence future decision-making based on past experience. When a person obtains a positive result from a decision, he or she will tend to repeat it [32]. The students' conclusions using their own words are shown in Figure 6.



(a)

Translated (a):
The effect of the value of c: if $c +$, the positive value of y will be the same, and vice versa.
$c +$: shifted up
$c -$: shifted down



(b)

Translated (b):
If $c +$, then y is shifted up
If $c -$, then y is shifted down

Figure 6. Examples of translated students' conclusion of the effect of changing the value of c

Students began to understand the meaning of the changes in the value of c in the graph of $f(x) = a|x+b|+c$. Changes in the c value shifted the graph vertically. If $c > 0$, the graph was translated by c units up. In contrast, if $c < 0$, the graph was translated by c units down. This condition occurs because the existing schemes provide a quick and accurate summary of relevant information so that it helps someone to analyse and interpret problems [33]. At the end, the teacher asked the students to describe some of the absolute function graphs verbally without drawing compared to the graph of $f(x) = |x|$. The goal was to observe the level of students' understanding of the algebraic representation of the absolute function of the general form $f(x) = a|x+b|+c$ in graphical form. Providing multiple representations in learning for students will improve students' ability to understand functions.

The results of the study show that, with the strategy of delayed scaffolding using GeoGebra, students were encouraged to analyse the effect of changes in values of *a*, *b*, and *c* in absolute functions. Students were stimulated to understand absolute functions in the form of symbolic expressions and graphics, expressing their understanding verbally. The use of technology has proven to improve the achievement of learning outcomes in mathematics classes. Reasoning is a process of making conclusions that teachers must always strive for in the practice of teaching mathematics in classrooms [28]. Through reasoning using multiple representations, students experience the process of obtaining their own understanding, which is essential for learning future algebraic topics.

4. Conclusion

The timing of implementing the scaffolding by the teacher in the learning process is essential. The delayed scaffolding using GeoGebra helped students to understand the translation of the representation of absolute functions. Thus, the delayed scaffolding using GeoGebra is an effective strategy in learning mathematics. The teacher provided time for the students to explore various graphs from various symbol expressions of absolute functions, then asked them to deduce from the graphical representation of functions using their own words by writing in books and expressing their understanding orally. The students' conclusions were discussed in class to obtain the correct conclusions, guided by the teacher. This strategy improved these students' ability to understand the translation of representations from symbolic forms to absolute function graphs.

The topic of the implementation timing of the scaffolding strategy in the teaching practices, especially mathematics, still needs to be explored further. Future work will analyse how teachers manage the time for providing scaffolding and choose the right scaffolding tools in different situations of mathematics learning.

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