Visual Understanding of Problem and Pictures´ Occurrence in Educational Process

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Abstract **– In this paper we describe functions of pictures and their using in educational process. We focus specifically on their occurrence in mathematics and economics. We present these functions of pictures in application and non-routine problems. Visualization of theme with using pictures is an aid for a better understanding of each problem in different subjects. From the point of view of the given topic, we have also analysed the math textbooks.**

Keywords **– Functions of pictures, Textbooks, Problem solving, Mathematics, Economics, Slovakia.**

1. Introduction

People have unlimited ability to remember, but have only limited active mind that at learning cannot exceed. If the students learn some abstract concept, considerable part of their active memory capacity is spent on figuring out the given situation which the teacher explains. [1]

Author in [2] has argued that visualization, as both the product and the process of creation, interpretation and reflection upon pictures and images, is gaining increased visibility in mathematics and mathematics education.

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Creating visual representations for math students can open up their understanding so in the article we are paying attention to the picture material which is more interesting for the students, then the relieved difficult interpretative or professional text. It is helpful to provide adequate pictorial ideas to students, fleshing out the specific problem and offering instructions to follow in a given solution.

2. Analysis of the present situation in secondary education in Slovakia

National Program of Education is official document for all Slovak schools in which are included the main goals and requirements of the math education. This program (named ISCED 2 and 3A) specifies standards for the lessons of mathematics in Slovak schools. [3]

Mathematic textbooks inherently belong in school education. These textbooks must comply with educational program and authors would respect their didactic principles in their creation.

The principle of illustration requires that the curriculum is inspired by situations students are familiar with. Specific examples are properly complemented with pictures, sketches, tables and more graphical interpretations. Explanations of the lessons are then closer to the students and they can create accurate and permanent concepts.

We analyzed mathematic textbooks released before the reform (books published in the year 2008) and after reform from the perspective of pictures in text and tasks with specified image, see [4],[5]. These types of textbooks are still used in the teaching process.

Images in these textbooks were related to mediating the teaching process, have incentive character and also fulfill cognitive and aesthetic significance. They were part of a few award tasks. Specified images were part of mainly issues with geometric focus.

In textbooks [5] we can see an emphasis on students' understanding because they are

predominate with assignments and drills (23,1%), assignments specified in image (12,2%) and exemplarily solved examples and the graphic solution (10,8%).

The teachers in secondary education should know how to represent abstract concepts with different methods. In practice, most often is used graphic explanation. This ability need to be developed in the future for teachers on the universities. The teachers are reliant on own ideas or on pictures in textbooks, which are not always suitable.

Representation within teaching used to have and will have key position. Using pictures in the learning process requires a good knowledge of the content and scope of the concept or a problem. In addition to a sense of quantity and frequency representation, and frequent rendering of development and abstraction and shortcomings connected with pictures doesn't hold back and ends with deficit of understanding. [6]

The following image list processes of processing visual Information and their transfer to student memory and subsequent use of information in continues education. [7]

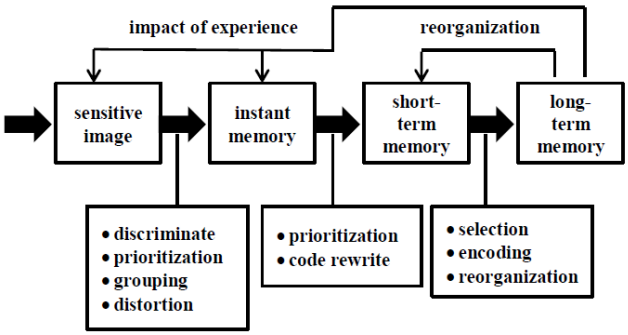

Figure 1. Working with an image [7]

3. Functions of picture and their use in mathematical problem solving

In the following parts we will specify the function of images, which ones we reached after the analysis in available didactic materials and textbooks. We can separate them as following:

- illustration function of the picture (in Table 1. as A),
- mentally-summarizing function of the picture (in Table 1. as B),
- picture as a tool for the teacher to get better formulation of the problem or hypotheses (in Table 1. as C),
- picture as a tool for teacher, to easy interpretation of learning curriculum (in Table 1. as D). [8]

Illustrative image function is needed on representations of features and concepts, such as pictures can be included in the classroom process in all learning stages. Students can link the specific mathematical entries with specific visual memorizing. This way connected contexts are stored in a long-term memory.

Image related to mental-summarizing function are summary and information that are related to the given mathematical problems, serves mainly to better remembering the terms or properties. Its importance we see especially in consolidation and deepening of a particular subject.

Pictures, which can be tools to search different formulations or hypotheses in mathematical issues, we use at implementation concepts. So, introduction concept related to definitions and allegations of relation to which ones need to pronounce or to prove, while images are an integral part of this process.

Teachers often extend using illustrative image - therefore to visualize the problems (we mention this in introduction). Students with help of illustrative image are faster acquiring new knowledge, and interpret solutions to mathematical problems.

Many researchers [9], [10], [11], [12], [13], [14], [15],[16] consider imagistic representations as a fundamental cognitive system for mathematical learning and problem solving. Expert mathematicians as well as mathematics students perceive visual representations as a useful tool in mathematical problem solving and frequently attempt to use them [17], [18], [19]. Authors in [18] proposed five functions of pictorial representations: decorative, representational, organizational, interpretational and transformational. Others see [20] proposed a similar categorization for the functions of pictures in mathematical problem solving: decorative, auxiliary-representational, auxiliary-organizational and informational. According to the above studies [17], [18], [20]:

- Decorative pictures do not provide any actual information concerning the solution of the problem.
- Auxiliary-representational pictures represent part or all of the problem content, but are not necessary to be used in order to solve the problem.
- Auxiliary-organizational pictures help the students to solve the problem by guiding them to organize the given statements of the problem.
- Informational pictures provide information that is essential for the solution of the problem. The problem is based on the picture.

Some researchers have found [10], [21], [22] that visual representations promote problem solving success. According to [23] students who are made aware of mental representations built on visual

imagery perform better and change their convictions related to mathematics. On the other hand, the results of [13] research indicate that the presence of both the representational and the decorative picture did not have a significant impact in pupil's performance, even though pupils' attitude towards them is positive.

In [24] author said that picturing a problem often is the key to helping students understand the problem and identify a solution.

We can say that pictures or visual representation of the problem are important in problem solving. More powerful strategy for students, which can help understanding given problem, is making pictures.

4. Pictures in Slovak math textbooks

Textbooks are important factors in influencing mathematics education. Examples in them can help students to acquire correct problem solving including the methods. We will focus on the analysis of mathematics textbook from the point of view of the given issue.

We selected math textbooks for students within age 13-14 years and the pictures in them, which are related with added functions. Students in this age often still work with pictures material. They use textbooks, variety pictures and colored compositions in the books. Also, they are using graphic solutions.

In Table 1. we are shoving overview of thematic units, which are in the textbooks with all of the related pictures. More can be seen in [25].

Table 1. Pictures in math textbooks

Part of curriculum / functions of picture	*A*	*B*	*C*	*D*
We remember…	22	33	33	7
Fractions	61	52	32	39
Cube and rectangular prism	33	49	35	28
Combinatorics	11	2	3	14
Rational numbers	12	5	19	6
Surface area and volume of cube and rectangular prism	22	28	15	8
Percent	18	29	16	22
Ratio, map scale and plan	13	27	21	12
Direct and inverse proportion	12	30	15	8

From the following findings, it is obvious that the picture materials are important for teaching mathematics. It is beneficial for teaching and helpful from different aspects according to the already explained functions. From Table 1. we can see that every function has representation in the used mathematic textbooks. There are parts like geometry where the pictures are expected. Explanation and solution of the geometric problems are not possible without pictures. But fractions or percent are follow-up with concrete pictures with variety functions. So we can state from this textbook analyzed facts, that the pictures have foundation in every teaching system with variety usage. We are bringing examples of concrete tasks, from mathematic textbooks with sample functions:

- illustration function of the picture (part of curriculum: Direct and inverse proportion),

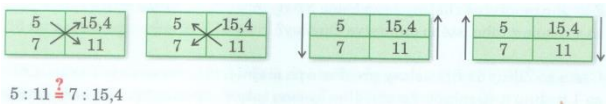

Figure 2. Example of using picture from math textbook for the class 7 (Part 2) in Direct and inverse proportion

- mentally – summarizing function of the picture (part of curriculum: Percent),

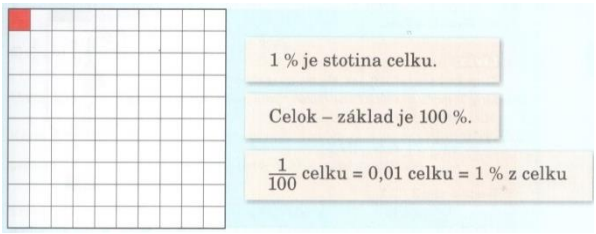

Figure 3. Example of using picture from math textbook for the class 7 (Part 2) in Percent

- picture as an aid to find a formulation (part of curriculum: Cube and rectangular prism),

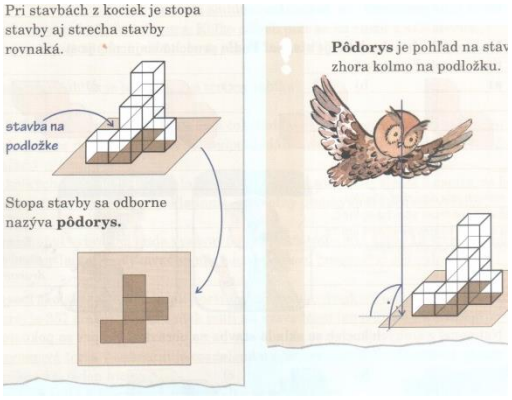

Figure 4. Example of using picture from math textbook for the class 7 (Part 1) in Cube and rectangular prism

- picture as a tool in improved interpretation of learning (part of curriculum: Pie chart).

Figure 5. Example of using picture from math textbook for the class 7 (Part 2) in Pie chart

In this part we will present individual functions of picture on specific mathematical problems.

Display of illustration image function is mentioned in Figure 6. There is explaining central symmetry as direct identity. We can use this picture as a link of the mentioned concepts (central symmetry – direct identity) adopted by exposition part of learning process. Student is gaining correct idea about concepts and adopts the planed learning.

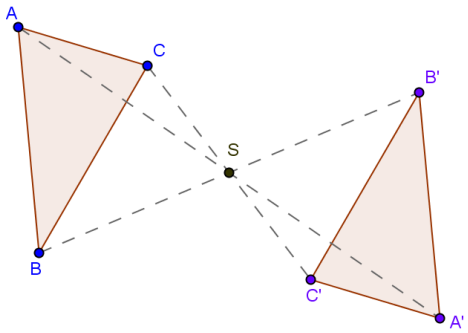

Figure 6. Central symmetry

Picture 7. is related to mental-summarized function. Functions marked in the given picture are mutually inverse and the picture represents relations between these functions. As we can see the inverse functions are symmetric along the axis $y = x$. Students can easier remember the concepts and properties of these functions.

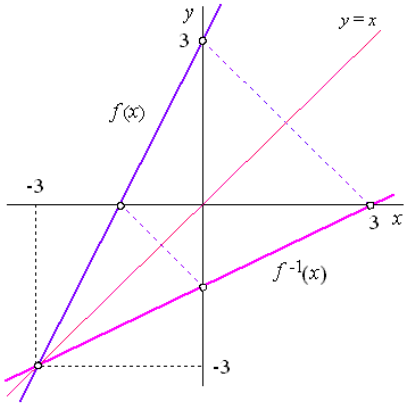

Figure 7. Inverse functions

Another Figure 8. can be used in establishing the volume of bodies. And it is in this case the image is an integral part of the issue. The image is part of the claim that the volume of the pyramid is one third the volume of the prism. The formula for the volume of any prism is known as $S.v$, where S is the content of the base of the prism and v is the height of the prism.

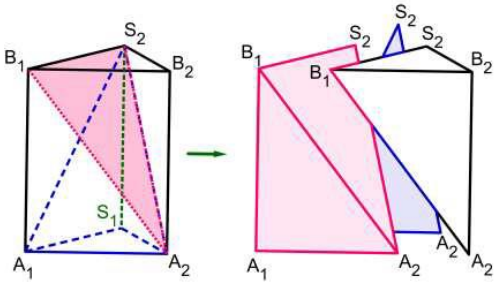

Figure 8. Volume of the prism [26]

Figure 9. is an example of counting fractions, it is $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$. Interpreted to illustrate operations with fractions and how they work, each fraction is also some graphical interpretation.

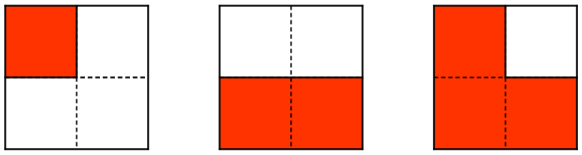

Figure 9. Counting fractions

5. Picture in economic application and non-routine problems

Pictures have an important function not only in mathematics. In this part we will deal with economic application because economics is closely associated with mathematics.

Pictures have wide application in economics. For example, Figure 10. means representations of concepts of budget line and budget constrain (blue triangle ABC). We can also watch new budget line in case that government allows the consumer to get free food (line 1) or government provides the consumer definite sum of money per month (line 2) or government allows the consumer food stamps (unlimited, for example they can buy food worth 6 € for 4 €, line 3). These situations increase consumption of food, change income of the consumer, and decrease the absolute slope of budget line. Creating visual pictures helps students think through mathematics and interpret as their thoughts.

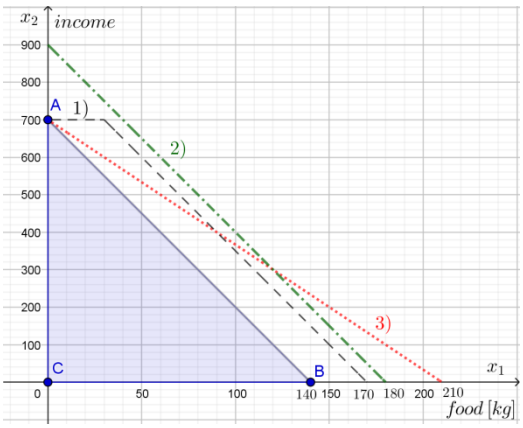

Figure 10. Presentation of the necessary concepts

For the company to make a profit, it's obvious that the revenue must be higher than the cost. Figure 11. summarizes main relationships between profit, costs and revenue. We can use it to better remember the terms and relationships between them.

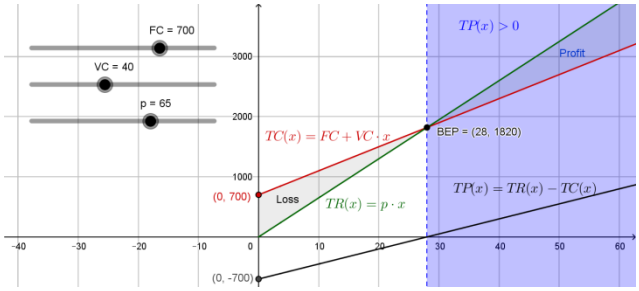

Figure 11. Summarizing relationships

The area between two curves is the integral of the absolute value of their difference. This is one of the lots of applications of integral calculus. The pictures are used in those types of tasks not only for illustration but also as a tool for finding a solution. In this note we look at the importance of willingness to pay for different goods and services. Consumer and producer surpluses are relatively easy to calculate if the supply and demand curves are straight lines. However, in realistic models of the economy, supply and demand generally do not to behave in this way. So we can calculate the area between two curves by the definite integral and we can find the solution through the picture (see Figure 12.).

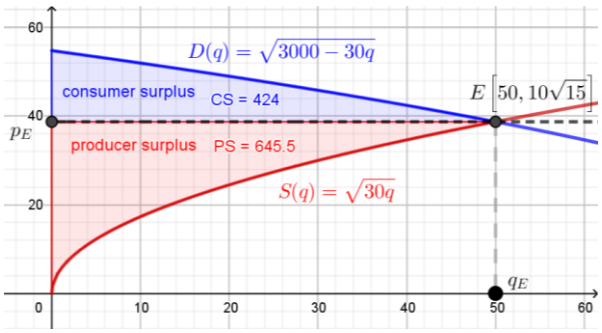

Figure 12. Solution of problem through the picture

The pictures are very helpful during explaining how a tax affects market participants or how deadweight loss and tax revenue vary with size of a tax.

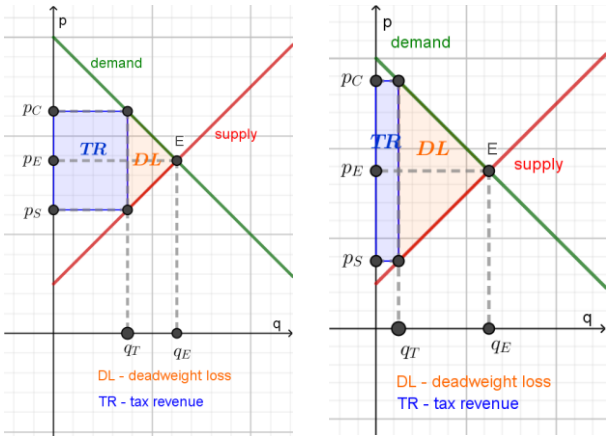

Figure 13. Formulations of given problem through the picture

The pictures are helped to find formulations (see Figure 13.): as the tax rate increases, its dead load is rising rapidly but because the size of the market has considerably diminished, the tax brings little income.

6. Conclusion

Visual representation of special problems can also be understood as a cognitive tool important to achieve a higher quality of visual thinking of students and their ability to apply complex mathematical knowledge. Application problems teach students connect mathematical knowledge with practice and allow to development of interdisciplinary thinking of them. Through graphical tasks´ interpretation, we link the aspect of the object illustration with the aspect of the representation and the description of any phenomenon.

In mathematics, it is possible to use different images, which proves that there is room on the scope of each type of feature images and forms of processing, similarly, in each of the learning process.

We can say that all functions of pictures were not only useful, but also essential and substantial in problem solving. The use of the pictures can lead students to decisions, which in turn enabled them to find correct solutions for the problems.

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