Dynamic Behavior of a Cylindrical Shell with a Liquid under the Action of Nonstationary Pressure Wave

Boris A. Antufev [1], Vasiliy N. Dobryanskiy [1], Olga V. Egorova [1], Eduard I. Starovoitov [2]

[1] *Moscow Aviation Institute (MAI), Volokolamskoe Highway, 4, 125993, Moscow, Russia*
[2] *Belarusian State University of Transport, Kirov Street, 34, 246653, Gomel, Republic of Belarus*

***Abstract* – The problem of axisymmetric hydroelastic deformation of a thin cylindrical shell containing a liquid under the action of a moving load is approximately solved. It is reduced to the equation of bending of the shell and the condition of incompressibility of the liquid in the cylinder. The deflections of the shell and the level of lowering of the liquid are unknown. For solution, the Galerkin method is used and the problem is reduced to a system of nonlinear algebraic equations. A simpler solution is considered without taking into account the incompressibility condition. Here, in addition to the deformed state of the shell, the critical speeds of the moving load are determined analytically.**

***Keywords* – cylindrical shell; hydroelastic axisymmetric deformation; liquid; incompressibility condition; lowering the liquid level; moving load; quasistatic solution; critical speed.**

1. Introduction

In liquid rockets, for the correct dosage of refuelable fuel components, it is necessary to take into account the bending of the walls of the tanks and as a result of lowering the level of the free surface of the liquid there. During their operation, the action of the pressure wave distorts the preliminary deformed state of the walls of the tanks, which in turn affects the critical velocities of this wave. All of these facts lead to the need to solve the hydroelastic problem for a cylindrical shell, taking into account the incompressibility of the liquid and the action of the moving load [1], [2]. A feature of this hydroelastic problem is the influence of the deformed state of the tank walls on the hydrostatic load, which is determined in accordance with Pascal's law taking into account the incompressibility of the liquid. As a result of this, the deflections of the shell are already nonlinearly dependent on the load, in contrast to the usual problems of static strength of tanks. The following considering problem is further complicated by taking into account the influence of a pressure wave on the walls of the shell, considered as the action of a moving radial load moving at a constant speed along its lateral surface [3], [4]. Therefore, in order to solve the problem, in addition to using the shell equations under the action of hydrostatic pressure and moving load, it is also necessary to take into account the condition of incompressibility of the liquid, which introduces nonlinear effects into the solution. The neglect of this condition greatly simplifies the problem, transferring it from the hydroelastic area to the usual static problem of shell strength. In the proposed problem, in addition to studying the lowering of the liquid level, another important problem was solved: determination of the critical speed of the moving load, upon reaching which a loss of stability of the shell walls is possible, which threatens the destruction of the entire structure of the aircraft [5], [6]. From the point of view of practice, the minimum critical velocity is the most interesting issue, the expression for which in a closed form was obtained in this work.

DOI: 10.18421/TEM102-39
https://doi.org/10.18421/TEM102-39

Corresponding author: Boris A. Antufev,
Moscow Aviation Institute (MAI), Moscow, Russia.
Email: antufev_boris.a@mail.ru

Received: 21 September 2020.
Revised: 06 April 2021.
Accepted: 12 April 2021.
Published: 27 May 2021.

2. Methodology

The solution to the hydroelastic problem is based on a change in the level of the free surface of the liquid in the cylinder due to deformation of its walls. It should be noted here [7], [8]. For this, the resolving system of equations includes, in addition to the equation of axisymmetric bending of the cylinder under the influence of gyrostatic pressure and a movable radial load, also the condition of incompressibility of the liquid [9], [10]. Lowering the liquid level leads to a change in the hydrostatic load and, as a consequence, to the nonlinearity of the problem. Application of the Galerkin method reduces the problem to a nonlinear system of algebraic equations, the solution of which has purely computational difficulties [9], [10]. Therefore, a simpler version of the solution of the problem is proposed that does not take into account the incompressibility of the liquid. In this case, application of the Galerkin method reduces the problem already to a system of linear algebraic equations. In both variants of the solution, the problem of determining the critical speed of the load at which the loss of stability of the shell walls occurs, as well as the effect of the liquid on this speed, is investigated [11], [12].

3. Results

We consider hydroelastic deformation of a thin cylindrical shell containing a liquid under the action of a pressure wave. We will represent this wave as an infinite uniformly distributed radial load moving along the side surface of the shell with a constant speed V. In Figure 1, for definiteness, a shell is shown freely supported on both ends with an absolutely hard bottom. We believe that in an undeformed state it is completely filled with a liquid with a specific gravity \gamma, and in the curved form of its walls, the height of the liquid column is equal to H (Figure 1).

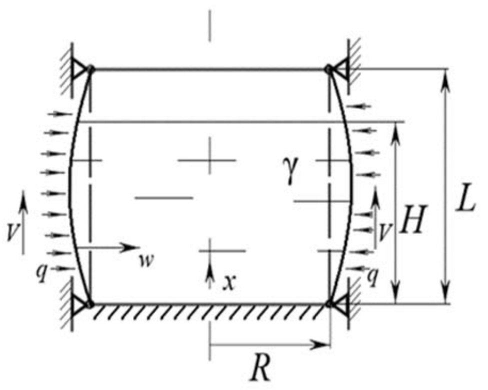

Figure 1. Absolutely hard bottom cylindrical shell

The cylinder is in an axisymmetric deformed state characterized by its normal displacements w. In addition to the hydrostatic pressure determined in accordance with Pascal's law, a moving load of intensity q, which in turn consists of static and inertial components, also acts on the walls of the shell. We solve the problem in a quasistatic setting, according to which the deflections of the cylinder w depend only on the longitudinal coordinate x and do not depend on time t [13]. Then, assuming that over time t the load element passes the distance $x = Vt$, the total load on the shell from the action of the pressure wave will be the following:

$$q + \frac{q}{g}\frac{\partial^2 w}{\partial t^2} = q + \frac{q}{g}V^2\frac{\partial^2 w}{\partial x^2}, \tag{1}$$

where g is gravitational acceleration. The deformed state of the shell is described by the equation of its axisymmetric bending under the influence of hydrostatic and mobile loads:

$$\frac{d^4 w}{dx^4} + 4\beta^4 w + \frac{q}{gD}V^2\frac{d^2 w}{dx^2} = -\frac{\gamma}{D}(H - x) + q, \tag{2}$$

where

$\beta^4 = 3(1-\mu^2)/R^2h^2$, $D = Eh^3/12(1-\mu^2)$, R and h are the radius and the thickness of the shell, E and μ are the elastic modulus and the Poisson's ratio of its material, respectively. The condition of incompressibility of the liquid in the shell has the following form [1]:

$$\int_0^{2\pi}\int_0^H wRdxd\theta = \pi R^2(L - H), \tag{3}$$

where θ is the circumferential coordinate. Given the axisymmetric deformation of the shell, relation (3) is simplified:

$$\int_0^H wdx = \frac{R}{2}(L - H). \tag{4}$$

The value of $L - H$ indicates the level of lowering of the free surface of the liquid in the cylinder. In the system of equations (2) and (4), the unknown are the deflection function of the shell $w(x)$ and the new position of the liquid level after the deformation of H. To solve the problem, we use the Galerkin method, according to which we represent the deflection of the cylinder in the form of the following series:

$$w = \sum_{i=1}^{N} w_i\phi_i(x), \tag{5}$$

where w_i are the unknown coefficients, $\varphi_i(x)$ are the specified coordinate functions satisfying the boundary conditions at the ends of the shell. Applying the procedure of the Galerkin method to equation (2) and substituting series (5) in the relation (4), we obtain a system of nonlinear algebraic equations of the following form:

$$\sum_{i=1}^{N} w_i a_{ij} = -\frac{\gamma}{D}\int_0^H (H-x)\phi_j dx + q b_j$$
$$(j = 1,2,....N),\ \sum_{i=1}^{N} w_i b_j = \frac{R}{2}(L-H), \quad (6)$$

where

$$a_{ij} = \int_0^L \phi_i{}^{IV}\phi_j dx + 4\beta^4 \int_0^L \phi_i\phi_j dx + \frac{q}{gD}\int_0^L \phi^{II}\phi_j dx,$$
$$b_j = \int_0^L \phi_j dx. \quad (7)$$

The coefficients b_i we obtained from the coefficients b_j by replacing the index j by i. Unknown constants w_i enter the system of equations (6) linearly, and the constant H enter the system of equations nonlinearly. The latter circumstance sharply complicates the solution of the problem. In the case of free support of the ends of the shell (Figure 1.), taking in (5) $\phi_i = sin(i\pi x/L)$, the first member of the series makes the largest contribution to the deformed state of the cylinder [1]. Therefore, preserving only it from the incompressibility condition (4), we obtain the relation between w_1 and H in the following form:

$$H = (1 - \frac{4w_1}{L\pi^2}).$$

This formula in its structure coincides with the formula obtained when solving a similar problem. However, in this particular case, we only reduce the dimension of the problem by one, and the computational complexity of the solution remains because the coefficient w_1 nonlinearly enters into the equations.

The solution can be simplified if we neglect the decrease in the liquid level due to deformation of the cylinder walls. In this case, the incompressibility condition is satisfied identically since $H = L$. Therefore, in (7), the upper limit in the integral for the hydrostatic load will be L. As a result, the system of nonlinear algebraic equations (6) becomes the system of linear ones (8).

$$\sum_{i=1}^{N} w_i a_{ij} = -\frac{\gamma}{D}\int_0^L (H-x)\phi_j dx + q b_j$$
$$(j = 1,2,....N). \quad (8)$$

The coefficients a_{ij} are determined by formulas (7), and only N constants w_1 are unknown. If the approximating functions in (5) are chosen orthogonal, then the coupled system of equations (8) decomposes into N separate equations, the typical solution of which has the following form:

$$w_i = -\frac{\frac{\gamma}{D}\int_0^L (H-x)\phi_i dx + q b_i}{a_{ii}} \ (i = 1,2,.....N). \quad (9)$$

The system of equations (6) and (8) includes the velocity of the load V as a parameter. Its critical values at which shell stability is lost from (6) can be determined by enumerating the values of V, tracking for some of them a sharp increase in deflections. It is identified with the loss of stability at certain (critical) values of speed. From equations (8), the critical velocities are found from the condition that the determinant of this system is equal to zero:

$$det\left|a_{ij}(V)\right|_{N\times N} = 0. \quad (10)$$

If the approximating functions in (5) $\phi_i(x)$ are chosen orthogonal, then the critical velocities can also be found in closed form, equating to zero the coefficient a_{ii} in formula (9):

$$V_{CR}^2 = -\frac{gD(\int_0^L \phi_i^{IV}\phi_i dx + 4\beta^4 \int_0^L \phi_i^2 dx)}{q\int_{01}^L \phi_i^{II}\phi_i dx},$$
$$(i=1,2,.....N). \quad (11)$$

This speed is completely independent of the properties and volume of the liquid in the shell. Its minimum value corresponds to the simplest form of loss of stability determined by the $\phi = sin(\pi x/L)$ function. It is equal to:

$$V_{CR}^2 = \frac{gD}{q}(\frac{\pi^2}{L^2} + 4\beta^4 \frac{L^2}{\pi^2}).$$

Higher critical speeds will be realized with more complex forms of loss of stability determined by the $\phi_i = sin(i\pi x/L)$ functions for $i = 2,3,...N$.

If it is necessary to determine the natural frequencies of the system, we consider the limiting case $H=L$, while we assume the problem to be nonstationary. In this case, the full second derivative symbolizing the normal movement of the shell and the acceleration of the shell walls in the direction normal to its surface will have the form:

$$\frac{d^2w}{dt^2} = \frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x\partial t} + V^2\frac{\partial^2 w}{\partial x^2}.$$

As a rule, the second term in this expression is neglected, which greatly simplifies the task as it is the Coriolis acceleration, then the equation of motion of the problem instead of (2) will be (12):

$$\frac{\partial^4 w}{\partial x^4} + 4\beta^4 w + \frac{q}{gD}(V^2\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2}) = -\frac{\gamma}{D}(H-x) + q. \quad (12)$$

To solve this problem, we use Galerkin method. For this, the deflection of the shell is represented as follows:

$$w = sin\,\omega\, t \sum_{i=1}^{N} w_i \phi_i(x), \tag{13}$$

where ω is the vibration frequency, w_i are the unknown coefficients, $\phi_i(x)$ are the specified coordinate functions that satisfy the boundary conditions at the ends of the shell. Galerkin method procedure leads to equation (12). In matrix form, the resulting system of linear algebraic equations for the unknown coefficients w_i will have the following form:

$$[K - \omega^2 M]Y = B, \tag{14}$$

where K and M are square matrices of stiffness and masses of the shell, respectively, with dimensions $N \times N$, Y is the column of unknown coefficients in expansion (13) and B is the column of the right-hand sides associated with the hydrostatic load on the shell. The elements of these matrices and columns are of the following form:

$$K = [k_{ij}], \quad M = [m_{ij}], \quad Y = \{w_i\}, \quad B = \{b_i\}, \tag{15}$$

where:

$$k_{ij} = \int_0^L \phi_i{}^{IV} \phi_j dx + 4\beta^4 \int_0^L \phi_i \phi_j dx + \frac{q}{gD} V^2 \int_0^L \phi^{II} \phi_j dx\,, m_{ij} = \frac{q}{gD} \int_0^L \phi_i \phi_j dx, \tag{16}$$
$$b_i = \int_0^L \left[-\frac{q}{D}(H - x) + q \right] dx.$$

Next, it is necessary to determine the natural frequencies of the shell vibrations ω. For this, it is necessary to solve the system of equations (14):

$$det[K - \omega^2 M] = 0. \tag{17}$$

Equation (17) makes it possible to determine the lower part of the spectrum of natural frequencies in a one-term approximation. From here we get the following:

$$\omega^2 = \frac{k_{11}}{m_{11}}.$$

Using the dynamic criterion of stability, the critical velocity can be found from the condition that the natural frequencies of oscillations are equal to zero. Then putting $k_{11} = 0$ we get the following:

$$V_{KP}^2 = -\frac{gD(\int_0^L \phi_1^{IV} \phi_i dx + 4\beta^4 \int_0^L \phi_1^2 dx)}{q \int_{01}^L \phi_1^{II} \phi_1 dx}.$$

4. Conclusions

As the obtained solutions show, taking into account the liquid incompressibility nonlinearly affects both the critical speeds of the moving load and the lowering of the liquid level in the shell. The proposed solution is based on reducing the hydroelastic problem for the shell under the influence of hydrostatic pressure and a moving load to a joint system of one differential and one integral equation for the deflection of the walls of the shell and lowering the liquid level in it, which contains the velocity of the moving forces as a parameter. Moreover, the integral condition of the incompressibility of the liquid subsequently introduces nonlinear effects into the solution. The use of the Galerkin method for solving the initial relations nevertheless leads to a system of nonlinear, but already algebraic equations for the deflections of the shell. It is possible to get rid of nonlinear dependences only by neglecting the hypothesis of incompressibility of a liquid. But in this case, the hydrostatic and dynamic parts of one task break up. The action of a moving load does not affect the lowering of the liquid level, although it allows obtaining formulas for the spectrum of critical velocities. Moreover, for the most important lower critical velocity, this formula is obtained in a closed form. On the other hand, the solution of the - hydrostatic problem does not affect the determination of critical velocities, and, consequently, the loss of stability of the shell, which now depends only on its stiffness properties. From this there is the conclusion that the division of one complex task into two relatively simple ones leads to incorrect results. Therefore, to obtain a reliable solution, the proposed method for studying the problem seems to be the most optimal one.

Acknowledgments

This work was financially supported by the grants of the Russian Foundation for Basic Research No. 19-08-00579 and No. 19-01-00675.

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